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WAVE FORCES AND MOMENTS ON A SUBMERGED ELONGATED BODY OF REVOLUTION

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by

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SUBMITTED IN PARTIAL FULFILLMENT OF THE
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and

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Submitted to the Department of Naval Architecture and Marine Engineering on May 20, 1961 in partial fulfillment of the requirements for the Master of Science degree in Naval Architecture and Marine Engineering and the Professional degree, Naval Engineer.

ABSTRACT

Laboratory measurements are made of the forces and moments exerted by water waves on a stationary, submerged, horizontal submarine shape and on two-dimensional circular cylinders at various relative depths and angles of incidence with respect to the wave crest.

Test conditions include separate cases of drag predominance and inertia predominance through variation in the ratio $\frac{u_{max}T}{D}$ or its approximate equivalent, the wave height to body diameter ratio, H/D.

Comparisons of the experimental force and moment signatures are made with those predicted by two separate theories:

1. The potential flow solution of Cummins.

2. A modification of the linear superposition of inertia and drag forces proposed by Morrison. In this modification the forces and moments are summed in increments of body length using drag coefficients determined in steady flow at the local Reynolds number and using average inertia coefficients over a full wave cycle determined by Keulegan and Carpenter.

All calculations were performed on an IBM 1620 digital computer and program statements are included.

Comparison of theory and experiment shows:

1. Both theories agree very closely for predominance of inertia forces.

2. Where inertial forces predominate the predicted phase relationship is excellent and the error between theory and experiment at maximum force has an average value of approximately 19% for the modified Morrison theory and 17% for the Potential theory.

3. For the case of significant or predominant viscous forces the modified Morrison theory gives a very good prediction of phase relationships and yields an error in maximum force having an average value of approximately 22% for the modified Morrison theory and 32% for the potential theory.

4. Instrumentation difficulties are estimated to contribute heavily to the apparent error particularly with respect to measured wave height and vertical moment.

5. The true error between theory and experiment is believed due predominately to the use of inexact drag and inertia coefficients.

6. Additional experiments are needed to define the time variation of both the inertia and drag coefficients in cases of oscillatory flow.

Thesis Adviser: Peter S. Eagleson

Title: Assistant Professor of Hydraulic Engineering

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LIST OF SYMBOLS

A = cross-sectional area of body, ft^2

C = wave celerity, ft/sec

C_D = drag coefficient

C_m = inertia coefficient

D = diameter of the body (may be a function of axial distance), ft.

E = modulus of elasticity, lbs/in^2

$\vec{F}$ = total hydrodynamic force acting on a body (vector quantity), lbs.

$F_{( )}$ = component of total force in the x, y, or z (subscripted) direction, lbs.

$\vec{F}_I$ = inertial component of F, lbs.

$\vec{F}_D$ = drag component of F, lbs.

$F_{I_{( )}}$ = inertial component of force in the subscripted direction, lbs.

$F_{D_{( )}}$ = drag component of force in the subscripted direction, lbs.

F_o = maximum value of the forcing function, lbs.

G = modulus of rigidity, lbs/in^2

H = wave height, double amplitude, ft.

I_p = polar moment of inertia, in^4

K = $2\pi/L$, wave number

L = wavelength of water wave, ft.

M_f = mass of fluid displaced by the body, slugs

M_f' = virtual or added apparent mass, slugs

M_z = hydrodynamic moment about the y axis of the body (vertical moment), ft.-lbs.

M_y = hydrodynamic moment about the z axis of the body (lateral moment), ft.-lbs.

M_v = moment measured by the bending gage, ft.-lbs.

T = wave period, sec.

V = volume of the body, ft^3

C_2 = damping coefficient for square low damping

C^* = equivalent linear coefficient of damping

C^*_{free} = equivalent linear coefficient of damping for free vibration.

C^*_{forced} = equivalent linear coefficient of damping for forced vibration.

d = distance between axis of model and center of bending gage, ft.

d_i = inside diameter, in.

d_o = outside diameter, in.

$_f$ = subscript denoting quantity relating to the fluid.

g = gravitational constant, 32.178 ft/sec^2

h = depth of water from still water to the bottom, ft.

k = spring constant of the system, lbs./in.

ℓ = body length, ft.

m = mass of the vibrating system, slugs

$\vec{q}$ = wave particle true velocity (vector quantity), ft/sec.

$_q$ = subscript refers quantity to actual velocity vector.

t = time, sec.

u = fluid velocity component in the axial (x) direction, ft/sec.

v = fluid velocity component in the transverse (y) direction, ft./sec.

w = fluid velocity component in the vertical (z) direction, ft./sec.

$\bar{}_{max}$ = refers to the maximum value that quantity has attained during the wave cycle.

x = distance along the body axis, ft.

x_{forced} = displacement of the system in forced vibration, ft.

x_{free} = displacement of the system in free vibration, ft.

y = distance in the transverse direction in the horizontal plane, perpendicular to the body axis, ft.

z = distance perpendicular to the body axis in the vertical plane.

α = angle of attack, measured in the horizontal plane between the axis of the body and the direction of wave propagation, radians.

β = angle between vector g and the horizontal plane, radians.

γ = shear strain at the surface, in/in.

ϵ_1 = principal tensile strain, in/in.

ϵ_2 = principal compressive strain, in/in.

$\int^*_{free} = C^*_{free}/C_c$

$\int^*_{forced} = C^*_{forced}/C_c$

η = displacement of the free surface from still water, ft.

θ = phase shift of the system response, radians.

μ = Poissons ratio.

μ_x = component of a three dimensional doublet in the axial direction, ft^3/sec.

μ_y = component of a three dimensional doublet in the transverse direction, ft^3/sec.

μ_z = component of a three dimensional doublet in the vertical direction, ft^3/sec.

ν = kinematic viscosity, ft^3/sec.

π = 3.1416

ρ_f = density of fluid (water), $slugs/ft^3$.

σ = wave angular frequency = $2\pi/T$, sec^{-1}.

τ = shear stress at the outer surface of the tube, in/in.

ϕ_β = velocity potential describing body shape, ft^2/sec.

ϕ_T = total velocity potential = $\phi_W + \phi_\beta$, ft^2/sec.

ϕ_W = velocity potential of the water wave, ft^2/sec.

ψ = wave phase angle, radians.

ω_d = damped natural frequency of the vibrating system, sec^{-1}.

ω_n = undamped natural frequency of the vibrating system, sec^{-1}.

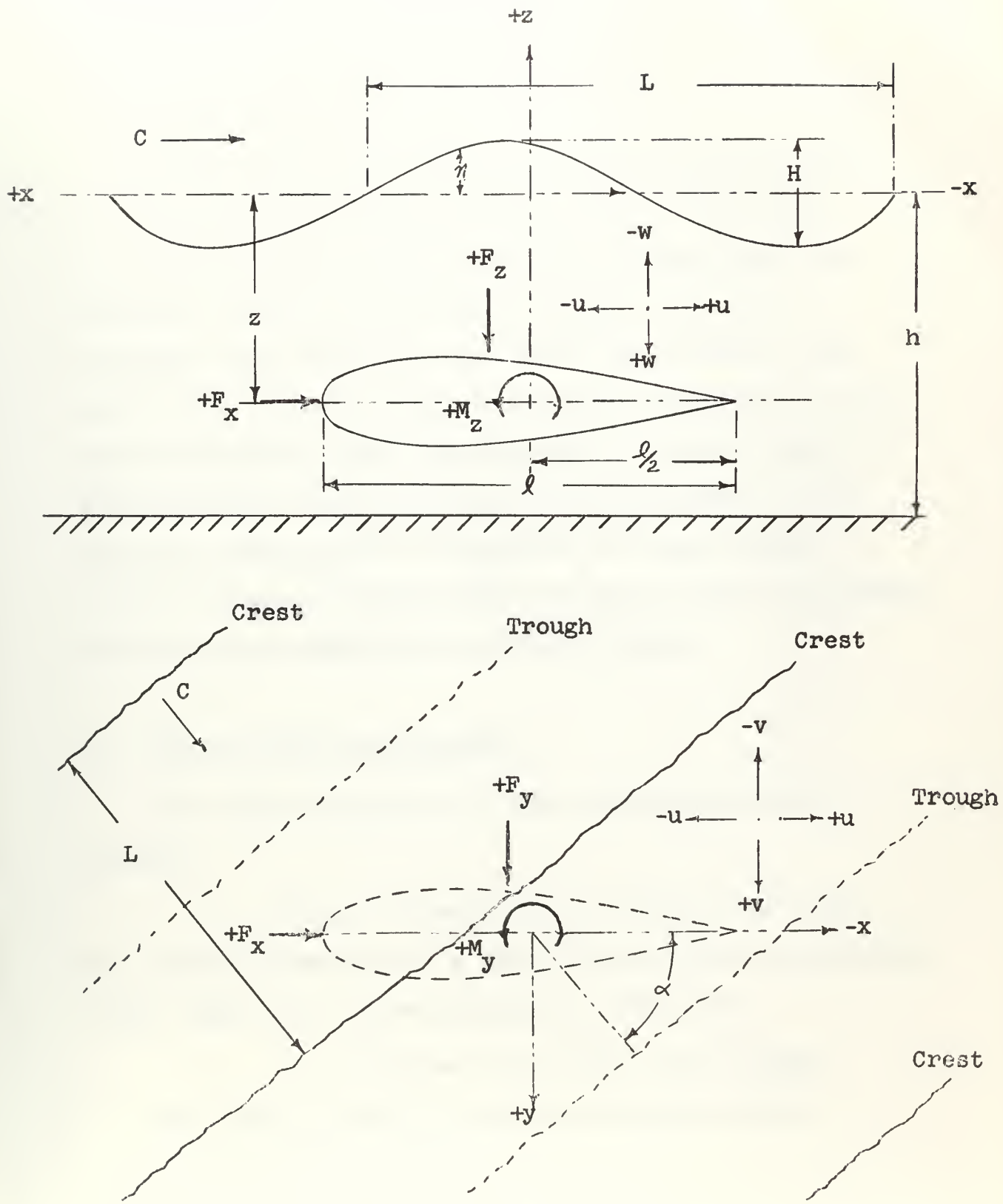


FIGURE 1. Definition Sketch

I. INTRODUCTION

A. Problem Statement

The problem of accurate prediction of forces and moments on a horizontal submerged body of revolution subjected to wave action has recently become quite important to several engineering groups. The naval architect has become involved with this problem in the case of the missile-carrying submarine operating at slow speed or hovering near the interface. Another quite unrelated group, those concerned with the design of offshore oil rigs and such structures on "Texas Towers" is also vitally concerned with the wave forces and moments on horizontal underwater structural members.

B. Scope of the Investigation

The basic objectives of this investigation are as follows:

1. Develop a formulation to predict the force and moment signatures of a body of revolution of arbitrary shape, under the following general conditions:

    a. Any given wave length, wave height, and depth of water (excluding breaking waves).

b. Any depth of the object below the still water level in which the interaction with the free surface is negligible.

c. Any given angle of attack of the axis of the body with respect to direction of wave travel.

d. Any diameter cylinder and any size or shape body of revolution submarine.

Because of the complexities involved in the theoretical formulations, high speed digital computer solutions are considered a necessity both from the standpoint of accuracy and evaluation time.

II. THEORETICAL CONSIDERATIONS

A. Force Theory

1. Summary of Previous Work

Considerable experimental and theoretical work has been done by other investigators concerning forces and moments on vertical pilings and various types of buoyancy elements. Outstanding among these studies is the work of Harleman (1),(2), Morrison (3), and Reid and Bretschneider (4). Little information of general utility is available for horizontal cylinders however.

The problem of wave forces on objects has been the subject of theoretical studies by Morrison (3), Cummins (5)(6), McCamey and Fuchs (7), and O'Brien (8). The first two of these theoretical approaches were adopted for application to the case of horizontal bodies:

(a) The Morrison theory (3) which utilizes the concept of a linear superposition of inertial and drag components to yield total force. Variations of this technique are most commonly used to predict the forces on vertical piles.

(b) A modified potential theory devised by Cummins (6) of David Taylor Model Basin to predict non-viscous forces and moments on a submarine moving beneath a wave train.

In the opinion of the authors, the Morrison technique offered the most accurate approach because of its inclusion (albeit approximate) of viscous effects. Thus the Morrison approach, with modifications, was adopted as the basic theory for this study.

The Cummins Potential theory is also applied in a parallel development primarily because of its value in checking the modified Morrison theory throughout the range from negligible viscous effects to cases where viscous effects are predominant.

2. Modified Superposition Theory

The Morrison theory (3) operates on the basic assumption that the total force, F, on a stationary submerged object subject to wave action, may be separated into independent inertial and drag components (F_I and F_D respectively) as described by Eq. (1)

$$F = F_I + F_D \tag{1}$$

It should be realized that this separation and assumed independence is an expedient and probably a fairly good one, but it is not strictly correct.

The inertial force is commonly divided explicitly into two parts:

a. The inertia of the accelerating fluid particles occuping the body volume but calculated in absence of the body.

b. An additional inertial force due to the modification in fluid acceleration caused by the shape of the solid body and its wake cavity.

The viscous or drag force as conventionally defined also represents the (implicit) summation of two terms:

a. The net surface resistance produced by the integral of the tangential stresses over the body surface.

b. The form resistance produced by the integral of normal stresses over the body surface and evaluated under steady state flow conditions for the free stream velocity of interest.

To the extent that variations in Reynolds number for a given acceleration produce appreciable changes in effective body shape and volume through variations in the boundary layer displacement thickness and the cavity

geometry; and to the extent that accelerations for a given Reynolds number affect the magnitude of the surface resistance, the inertial and drag forces as defined will be interdependent.

Any apparent variation in form drag due to the effect of acceleration on wake geometry may logically be considered an inertial force as all form drag truly is since the potential flow pressure distribution for the body plus wake cavity will incorporate all resistance save that due to tangential stresses.

Under the above assumption the inertial force may be written:

$$\vec{F}_I = M_f \, d\vec{q}/dt + M_f' \, d\vec{q}/dt \qquad (2)$$

Inertia of displaced fluid — Added inertia due to presence of body

in which M_f is the mass of the displaced fluid, M_f' is an additional apparent fluid mass, and q is the vector of instantaneous fluid velocity.

Thus

$$F_I = (M + M') \, d\vec{q}/dt = C_m M \, d\vec{q}/dt \qquad (3)$$

in which

$$C_m = 1 + \frac{M'}{M} \qquad (4)$$

and acceleration vectors (and also probably the rate of change of acceleration).

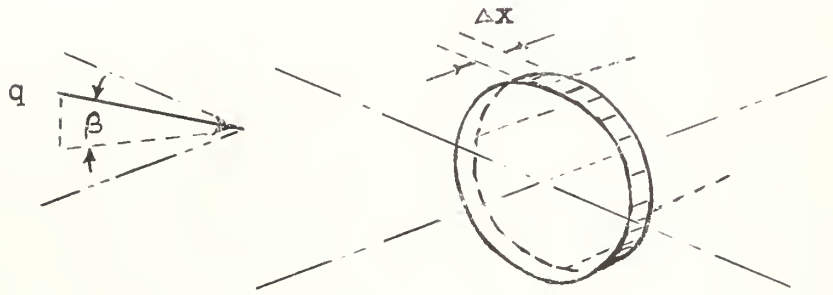


FIGURE 2. Typical Incremental Body Slice

The total drag force may be written as in equation (5)

$$\vec{F}_D = C_{D_q} \frac{\rho_f}{2} A_q \vec{q}^2 \tag{10}$$

in which the subscript q refers to variability of the quantity with the direction (and magnitude in the case of C_{D_q}) of the velocity vector, q.

If the angle of the velocity vector with respect to the horizontal plane is defined as β, then the true z component of the total drag may be written:

$$F_{D_z} = C_{D_q} \frac{\rho_f}{2} A_q q^2 \sin \beta \tag{11}$$

If we now assume that:

$$\vec{q}^2 \sin \beta \cong w^2 \tag{12}$$

then

$$C_{D_q} = C_{D_w}$$

$$A_q = \ell D$$

The drag force may be written in terms of the fluid velocity as:

$$\vec{F}_D = C_D \rho_f \frac{A}{2} |q| \vec{q} \tag{5}$$

in which the drag coefficient is a function of body shape, surface roughness and a body Reynolds number.

From equations (3) and (5) it can be seen that the incremental total force on an incremental slice of a body of revolution of diameter D may be described by:

$$\vec{\Delta F} = \vec{\Delta F}_I + \vec{\Delta F}_D \tag{6}$$

$$\vec{\Delta F} = \Delta F_x + \Delta F_y + \Delta F_z \tag{7}$$

where, for example:

$$\Delta F_z = \Delta F_{I_z} + \Delta F_{D_z} \tag{8}$$

The components of the inertial force may be written:

$$\Delta F_{I_z} = C_m \rho_f \left(\frac{\pi D^2}{4}\right) \frac{dw}{dt} \Delta x$$

$$\Delta F_{I_y} = C_m \rho_f \left(\frac{\pi D^2}{4}\right) \frac{dv}{dt} \Delta x \tag{9}$$

in which: a. x is always oriented along the axis of symmetry of the body.

b. C_m is determined by the body geometry and its orientation with respect to the total velocity

From the above, it can be written:

$$F_{D_z} = C_{D_w} \frac{\rho_f}{2} \ell D \, w^2$$

$$F_{D_y} = C_{D_y} \frac{\rho_f}{2} \ell D \, v^2 \qquad (13)$$

The approximation of equation (12) is acknowledged to be accurate only for two discrete phase angles of the wave motion. It was felt necessary nevertheless, to apply this approximation at all phase angles since empirical drag coefficients for the body shape "seen by the vector q" are not available. These coefficients are known however for the cylindrical shape presented to the component vectors w and v.

In keeping with this assumption, the C_{D_w} and C_{D_y} were obtained from circular cylinder data at Reynolds numbers based upon the respective component velocities w and y. Similarly the mass coefficients, C_m of equation (9) will be approximated through use of data from two dimensional circular cylinders and will thus be denoted by C_{m_w} and C_{m_v} respectively.

In a manner similar to equation (9), the components of drag force are now written:

$$\Delta F_{D_z} = C_{D_w} \frac{\rho_f}{2} \quad D \; |w| w \quad \Delta x$$

$$\Delta F_{D_y} = C_{D_y} \frac{\rho_f}{2} \quad D \; |v| v \quad \Delta x \qquad (14)$$

whereupon the vertical and lateral (perpendicular to the body axis in the horizontal plane) components of total incremental force may be written using equations (8), (9), and (14):

$$\Delta F_z = \left[C_{D_w} \frac{\rho_f}{2} D |w| w + C_{m_v} \rho_f \left(\frac{\pi D^2}{4}\right) \frac{dw}{dt} \right] \Delta x$$

$$\Delta F_y = \left[C_{D_v} \frac{\rho_f}{2} D |v| v + C_{m_v} \rho_f \left(\frac{\pi D^2}{4}\right) \frac{dv}{dt} \right] \Delta x \qquad (15)$$

The vertical and lateral moments (M_z and M_y) about axes through the geometrical center of the body may be obtained by multiplying equations (15) by the axial position measured from the body center:

$$\Delta M_z = \Delta F_z \, x$$

$$\Delta M_y = \Delta F_y \, x \qquad (16)$$

Then by summing these incremental forces or integrating over the length of the body, the total force may be obtained. However, these equations cannot be integrated directly because every term except ρ_f in equations (15) and (16) is a function of x, the position along the body axis.

It is further assumed that the average "free stream" particle velocity and acceleration to be used in the force

equations at a given section is the same as would be found at that section on the axis of the body but with the body removed from the flow. Note that all three of the velocity and acceleration components may be varying in magnitude and direction along the axis of the body at any particular instant of time.

The complete equations describing the variations of the three components of velocity and acceleration are given in Section II-B.

The coefficients C_m and C_D will vary with axial position on the body due to the variations in diameter, velocity, acceleration, etc., but the quantitative effect of these parameters is unknown. For the common case of steady flow about a cylinder, experiments have shown a well verified relationship between C_D and Reynolds number (Fig. 3). Likewise for the case of constant acceleration, theory and experiment have shown the inertia coefficient for a circular cylinder to be equal to 2.0. A thorough search of available literature revealed no information concerning instantaneous values of C_D and C_m in oscillatory flow. This is due to the fact that it is possible to separate the inertial and drag component of total force only at four discrete points in a wave cycle - points at which the velocity or acceleration go to zero

leaving only the inertia or drag components respectively. The only information found concerning the variation of C_m and C_D for circular cylinders in an oscillatory flow was that of Keulegan and Carpenter (9). These investigators correlated C_D and C_m with an acceleration parameter $\frac{U_{max}T}{D}$ which can be shown to be proportional to the ratio of drag to inertia forces. While both C_m and C_D were observed to vary over the wave cycle, a correlation of only the mean values of these coefficients with $\frac{U_{max}T}{D}$ was obtained.

Due to the lack of information concerning the instantaneous values of C_D and C_m the following approximations were adopted:

1. C_D is calculated at each point along the body from the conventional plot of steady state C_D vs local Reynolds number for two dimensional circular cylinders (Fig. 3). By this technique, C_D varies over the length of the body because both the diameter D, and the velocity of the Reynolds number vary with axial position on the body.

2. C_{m_w} and C_{m_v} are determined for respective values of period parameters, $\frac{w_{max}T}{D}$ and $\frac{v_{max}T}{D}$, from the plot provided in Keulegan and Carpenter (9) and reproduced in Fig. 4. The values of C_m in this figure are average values of C_m over the wave cycle for given values

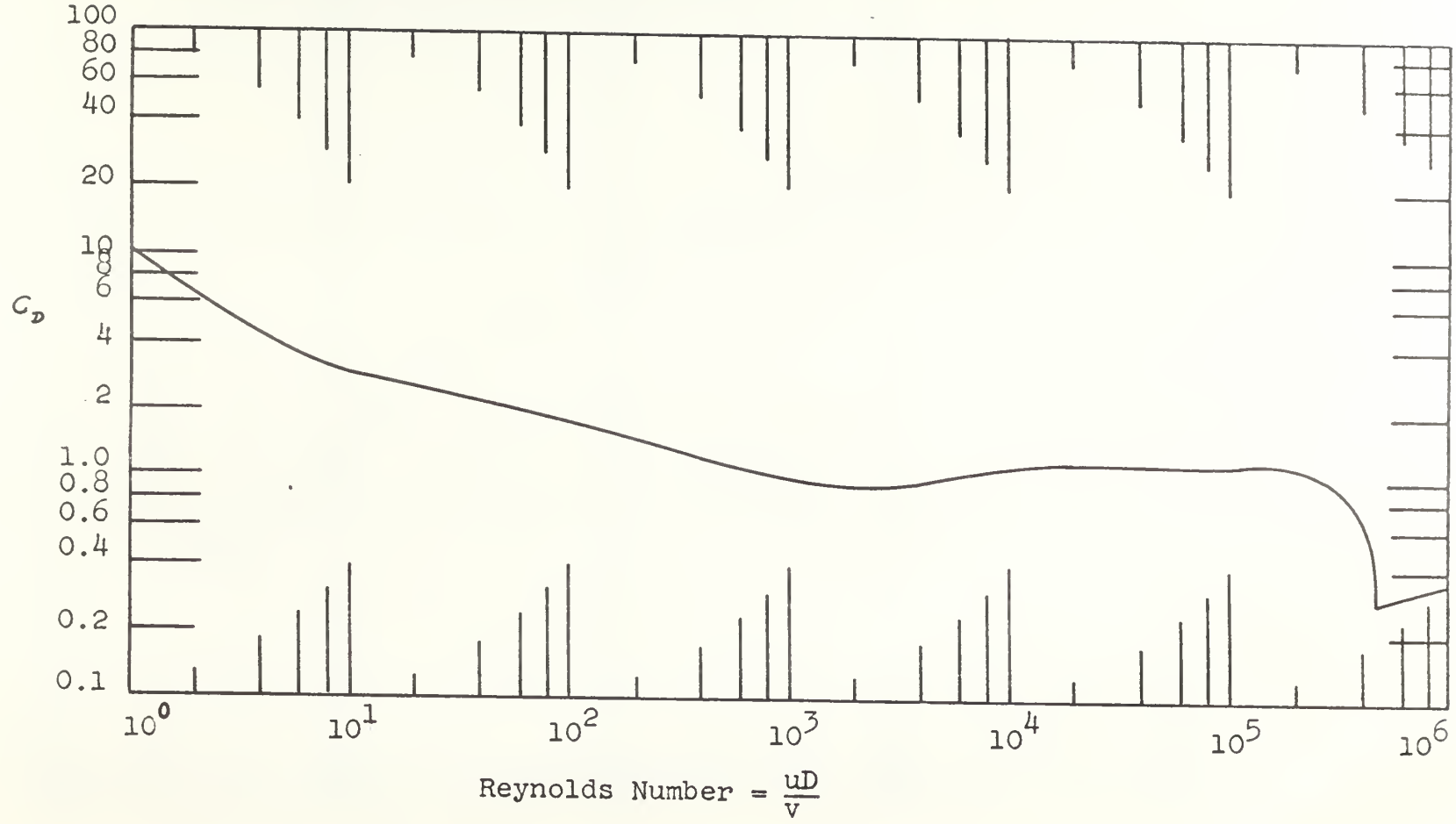


FIGURE 3. C_D vs Reynolds Number for Circular Cylinders

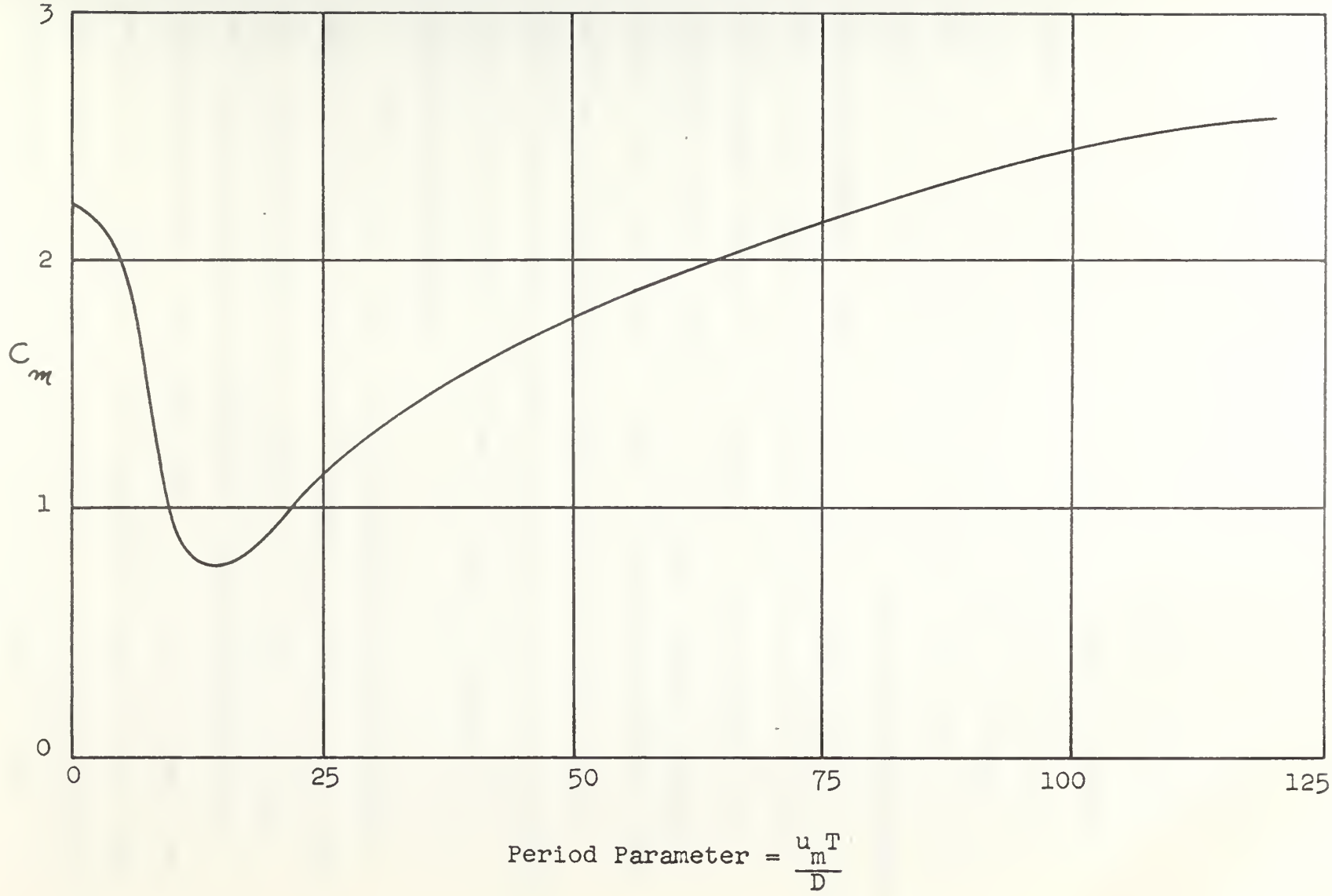


FIGURE 4. C_m vs $\frac{u_m T}{D}$ After Keulegan and Carpenter

of the period parameters. For a particular wave and a body of constant diameter along its length, the period parameter is constant and the average value of C_m is determined. For a body of varying diameter along its length, such as a submarine shape, the period parameter varies along the body and thus the average value of C_m varies with axial position along the body. As shown in Ref. (9), for a given period parameter, the instantaneous value of C_m does in fact vary. For some period parameters, the instantaneous values of C_m have a large deviation from the average value and therefore in these cases the use of the average values of C_m from Fig. 4 can introduce considerable error. However, it was felt that the average values of C_m, as shown in Fig. 4, did constitute the best information available and that the use of these values would provide better accuracy than the use of the constant value of 2.0 which is universally used in the case of constant acceleration.

With all the terms of the modified Morrison theory defined, the next step is to integrate equations (15) and (16) numerically over the length of the body. A simple rectangular numerical integration rule was employed in preliminary work. In later work, with an eye towards improving the accuracy of the integration while at the same time reducing the number of numerical integration stations, Tchebycheff's seven ordinate rule was adopted. The

superiority of Tchebycheff's seven ordinate rule is illustrated by results of a Bureau of Ships comparison of Simpson's rule with Tchebycheff's rule in which it was found that the latter rule was more accurate than the twenty station Simpson's rule. Besides being more accurate, the Tchebycheff rule is more economical of computer time since fewer ordinates are required for a given body length.

By way of summary, the following equations are used to represent the forces and moments acting on an elongated body of revolution in a train of waves:

$$F_z = F_{vertical} = \int_{-\ell/2}^{\ell/2} \left[C_D \frac{\rho_f}{2} D \, |w| w + C_m \frac{\rho_f \pi}{4} (D)^2 \frac{dw}{dt} \right] dx \tag{17}$$

$$F_y = F_{lateral} = \int_{-\ell/2}^{\ell/2} \left[C_D \frac{\rho_f}{2} D \, |v| v + C_m \frac{\rho_f \pi}{4} D^2 \frac{dv}{dt} \right] dx$$

$$M_z = M_{vertical} \int_{-\ell/2}^{\ell/2} \left[C_D \frac{\rho_f}{2} D \, |w| w \, x + C_m \frac{\rho_f \pi}{4} D^2 \frac{dw}{dt} x \right] dx \tag{18}$$

$$M_y = M_{lateral} \int_{-\ell/2}^{\ell/2} \left[C_D \frac{\rho_f}{2} D \, |v| v \, x + C_m \frac{\rho_f \pi}{4} D^2 \frac{dv}{dt} x \right] dx$$

3. Potential Flow Theory

The potential flow theory used in this report is an adaptation of the theory set forth by Cummins (6). This latter theory predicts axial force, lateral force and moment, and vertical force and moment on a slender body of revolution moving at a constant speed under a sinusoidal wave train. Since this investigation is confined to the case of a submarine shape or a cylinder with zero forward speed, the potential flow solution was simplified considerably.

As in all potential flow solutions, the assumptions must be made that the velocity field is irrotational and that a velocity potential ϕ_T does exist. It is further assumed that the velocity potential ϕ_T is composed of the sum of ϕ_W, the velocity potential of the wave without the body present, plus ϕ_β, the velocity potential of the doublet distribution which describes the profile of the body being investigated. The diameter of the body is assumed to be small compared to the wave length so that the flow which would exist in the volume displaced by the body, but with the body removed, may be considered approximately uniform. The strength of the three dimensional doublets describing the body are given as follows:

$$\mu_x(x) = -\frac{1}{16} D^2 u$$

$$\mu_y(x) = -\frac{1}{8} D^2 v \tag{19}$$

$$\mu_z(x) = -\frac{1}{8} D^2 w$$

From these doublet distributions, the forces and moments on the body are given as:

$$F_x = -4 \pi \rho_f \int_{-\ell/2}^{+\ell/2} \frac{d\mu_x}{dt} dx \tag{20}$$

$$F_y = -4 \pi \rho_f \int_{-\ell/2}^{+\ell/2} \frac{d\mu_y}{dt} dx \tag{21}$$

$$F_z = -4 \pi \rho_f \int_{-\ell/2}^{+\ell/2} \frac{d\mu_z}{dt} dx \tag{22}$$

$$M_y = -4 \pi \rho_f \int_{-\ell/2}^{+\ell/2} x \frac{d\mu_y}{dt} dx \tag{23}$$

$$M_z = -4 \pi \rho_f \int_{-\ell/2}^{+\ell/2} x \frac{d\mu_z}{dt} dx \tag{24}$$

The equations are quite a bit simpler than those of the modified Morrison theory but, of course, do not include any viscous forces. Solution of these equations has been carried out using both the Airy (sinusoidal, applicable in infinite depth) and Stokian (unsymmetrical, applicable in finite depth) wave theories.

B. Wave Theory

Early in the investigation, a decision had to be made as to which wave theory would best describe the sea wave which exerts the force on a hovering submarine or an offshore structure.

It should be pointed out that the sea wave exerting the force is not a regular wave but rather a statistical array of regular waves. Accepting the notation of superposition of harmonics, however, it is of value to study the regular wave. For the most practical range of wave lengths and water depths this regular wave is described by the Stokian theory.

The following equations as obtained from Ref. (10) define the profile, celerity and internal fluid particle velocities and accelerations associated with a Stokian wave:

$$\psi = (K x \cos \alpha + K y \sin \alpha + \sigma t) \qquad (25)$$

$$\eta = \frac{H}{2}\sin(\psi) - \frac{\pi}{8}\frac{H^2}{L}\frac{\cosh(kh)(2+\cosh(2kh))\cos(2\psi)}{\sinh^3 Kh} \tag{26}$$

Wave celerity:

$$C^2 = \frac{g}{K}\tanh(kh)\left[1 + \left(\frac{\pi H}{L}\right)^2 \frac{(5+2\cosh(2kh)+2\cosh^2(2kh))}{8(\sinh(kh))^4}\right] \tag{27}$$

Particle velocity:

$$\frac{u}{\cos\alpha} = \frac{v}{\sin\alpha} = \frac{\pi H}{L} C \frac{\cosh K(h+z)}{\sinh(Kh)}\sin(\psi) - \left(\frac{\pi H}{L}\right)^2 \frac{C}{\sinh^2(Kh)}$$

$$\left[-\frac{1}{2} + \frac{3}{4}\frac{\cosh 2K(h+z)}{\sinh^2 Kh}\right]\cos(2\psi) + \left(\frac{\pi H}{L}\right)^2 \frac{C}{2}\frac{\cosh 2K(h+z)}{\sinh^2 Kh} \tag{28}$$

$$w = \frac{-\pi H}{L} C \frac{\sinh K(h+z)}{\sinh(Kh)}\cos(\psi) - 3/4\left(\frac{\pi H}{L}\right)^2 C \frac{\sinh 2K(h+z)\sin 2\psi}{\sinh^4 Kh} \tag{29}$$

Particle acceleration:

$$\frac{1}{\cos\alpha}\frac{du}{dt} = \frac{1}{\sin\alpha}\frac{dv}{dt} = \sigma\frac{\pi H}{L} C \frac{\cosh K(h+z)\cos(\psi)}{\sinh Kh}$$

$$+ 2\sigma\left(\frac{\pi H}{L}\right)^2 \frac{C}{(\sinh Kh)^2}\left[-1/2 + 3/4\frac{\cosh 2K(h+z)}{\sinh^2 Kh}\right]\sin(2\psi) \tag{30}$$

$$\frac{dw}{dt} = \frac{\sigma\pi H}{L} C \left[\frac{\sinh K(h+z)}{\sinh Kh}\right]\sin\psi$$

$$-3/2\ \sigma\left(\frac{\pi H}{L}\right)^2 C \frac{\sinh 2K(h+z)}{\sinh^4 Kh}\cos(2\psi) \tag{31}$$

The equations describing the sinusoidal wave form also used are likewise presented here:

$$u = \frac{\pi Hc}{L} \cos\alpha \quad e^{2\pi z/L} \quad \sin\psi \tag{32}$$

$$v = \frac{\pi Hc}{L} \sin\alpha \quad e^{2\pi z/L} \quad \sin\psi \tag{33}$$

$$w = -\frac{\pi Hc}{L} \quad e^{2\pi z/L} \quad \cos\psi \tag{34}$$

$$\eta = \frac{H}{2} \sin\psi \tag{35}$$

$$c^2 = \frac{gL}{2\pi} \tag{36}$$

A qualitative plot (Fig. 5) of the variation of the velocity and acceleration for a typical water wave is included at this point as an aid to the reader in understanding the wave theory and the force theory.

C. Computer Program

1. General

A few brief comments concerning the computer programming problems and techniques should make the program listed in the appendix easier to understand. The nature of the equations developed to predict the forces and moments on a body of revolution invited the use of a high speed digital computer to perform the complex numerical integrations. A

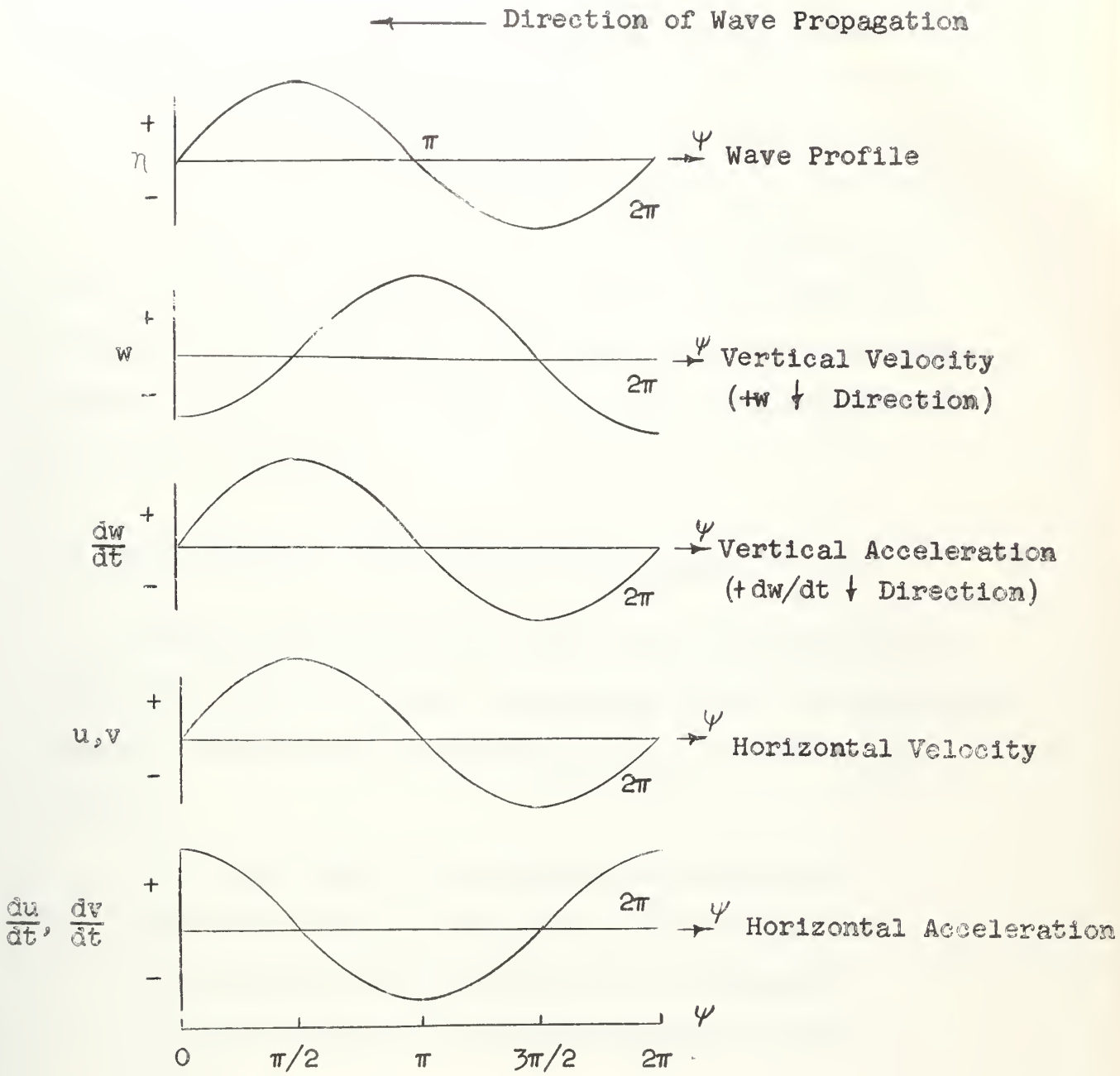


FIGURE 5. Qualitative Variation of Particle Velocity and Acceleration in a Water Wave

well designed computer program also provides added flexibility or generality to the solution and increases the reliability of the evaluated forces and moments.

The IBM 1620 computer of the Civil Engineering Department at M.I.T. was employed exclusively for all the work included in this report. It should be noted that this computer is much slower than the IBM 709 but was used because of the ready availability of computer time and because of the opportunity to become more familiar with operating procedures and programming techniques peculiar to this new, popular machine. A listing of the three main progams may be found in Appendix C.

2. Main Program - Modified Morrison Theory

Because the program was quite long and complex every effort was made to compute the various terms in an efficient manner. A few of the interesting aspects of the program follow:

a. All eight of the hyperbolic sine and hyperbolic cosine terms used in the Stokian wave equations are computed by one statement (Statement number 1, modified Morrison theory program).

b. The vertical and horizontal Stokian velocity and acceleration terms (Equations 28, 29, 30, and 31) are computed in one statement (Statement number 4).

c. The log-log plot of C_D vs Reynolds number (Fig. 3) was approximated by three polynomial equations (each covering a particular section of the curve) using a least squares technique of curve fitting. The use of three separate equations was necessary in order to obtain a good approximation of the curve with a low order polynomial. The curve of C_D vs Reynolds number as derived from Schlichting (11) and employed on the basis of the least square curve fit is shown in Fig. 3. Note that this curve was entered for both horizontal and vertical velocity components to determine corresponding horizontal and vertical C_D values.

d. In a manner similar to that described in c, the plot of C_m vs $\frac{U_{max}T}{D}$ was approximated by three polynomial equations. The original curve as derived from Keulegan and Carpenter is shown in Fig. 4.

e. It should be noted that a term called "Ratio" was printed out in the results. This term is equal to the total vertical drag force divided by the total force at that particular

time. This was inserted into the program so that the magnitude and direction of the vertical drag force component and the vertical inertial force component could be calculated if desired.

f. A special test was inserted in the program to check the hyperbolic sine term to insure that they were not getting too large for the computer. (Largest number allowed is 10^{+49}). Such a condition might arise for a deep water prototype wave.

g. In order to obtain the force and moment signature over a wave cycle, all three programs were designed to compute these values at eleven equally spaced stations corresponding to wave phase angles covering 0° to 360° in 36° increments. The results of each program as presented in Appendix A show a listing of these eleven groups of values of forces and moments.

3. Potential Flow Program

The potential flow programs have been listed in Appendix C and are straightforward and easy to understand.

Note that both the sine wave profile potential flow program and the Stokian wave profile potential flow program are much shorter and simpler than the modified Morrison program.

III. EXPERIMENTAL EQUIPMENT AND PROCEDURES

A. Experimental Program

An experimental program was selected to test three different areas of application of the modified Morrison theory:

1. inertial forces predominant
2. drag forces predominant
3. inertial and drag forces about the same order of magnitude

The ratio D/H, was used as a rough measure of the ratio of inertial to drag force. Typical values of D/H which corresponded to the three cases above were derived from Harleman's report on vertical cylinders (1).

In order to obtain more general information from the experimental program, two geometrically similar waves were proposed, the first being 10 ft. long and about 8 inches high, the second being 5 ft. long and 4 inches high. The use of a fairly large wave was deemed necessary in order to obtain measurable forces. The use of two wave lengths was intended to illustrate the effect of the ratio of body length to wave length on the measured wave forces and moments.

Based on the waves selected above and selected values of D/H, the following models were constructed for the experimental tests (Fig. 11)

1" diameter cylinder	30" long	aluminum tubing
2-7/8" diameter cylinder	30" long	aluminum pipe
4" diameter cylinder	30" long	plexiglass tubing
5.7" maximum diameter submarine shape	275" long	wood

Since the modified Morrison theory and its associated computer program was designed to compute forces and moments at any angle of attack and at any depth below the onset of surface effects, one model was tested at different depths and several models were tested at angles of attack of 45° and 90°.

Table I indicates the model type, orientation, depth, etc., for all of the experimental runs performed.

B. Experimental Equipment

Fortunately, instrumentation required to measure forces and moments, and laboratory wave making equipment was available in the M.I.T. Hydrodynamics Laboratory. This instrumentation was originally designed to measure forces and moments on vertical piles and various buoyancy configurations proposed for offshore oil rigs.

A brief description of the available equipment follows. A more detailed description of apparatus may be found in Ref. (1) and Ref. (2).

TABLE I. EXPERIMENTAL PROGRAM

Run No.	Model	Wavelength (ft.)	Approximate Wave Height (inches)	Depth of Body Below Still Water (inches)	Angle of Attack (α) (degrees)	Depth of Water (ft.)
1	4" cylinder	10'	7.5"	10	0°	2.0'
2	Submarine Shape	10	7.5	10	0°	2.0
3	2-7/8" cylinder	10	7.5	10	0°	2.0
4	1" cylinder	10	7.5	10	0°	2.0
5	4" cylinder	5	4.0	10	0°	2.0
6	Submarine Shape	5	4.0	10	0°	2.0
7	2-7/8" cylinder	5	4.0	6	0°	2.0
8	2-7/8" cylinder	5	4.0	10	0°	2.0
9	2-7/8" cylinder	5	4.0	14	0°	2.0
10	1" cylinder	5	4.0	10	0°	2.0
11	4" cylinder	5	4.0	10	45°	2.0
12	Submarine Shape	5	4.0	10	45°	2.0
13	2-7/8" cylinder	5	4.0	10	45°	2.0
14	Submarine Shape	5	4.0	10	45°	2.0
15	2-7/8" cylinder	5	4.0	10	45°	2.0

1. Wave Tank and Wave Generator

(Figs. 6 through 9) show the 90 foot wave tank with the piston type wave-maker at one end. The glass walled tank is 37" deep and 30" wide with a sloping beach wave absorber at the far end.

2. Test Stand (Fig. 10)

The test stand on which the force and moment gages were mounted was constructed of steel pipe and angles and was mounted on rubber pads for isolation against external vibration. It holds the force and moment transducers from which the test object is suspended by means of two slender, shielded rods passing through the free surface. The stand was securely anchored by steel guide wires for added rigidity.

3. Instrumentation

a. Horizontal Force Gage (Figs. 14 and 15 and Ref. 1). A highly sensitive Shaevitz linear differential transformer is utilized in this "Portal type" gage to yield a voltage output proportional to the horizontal force while remaining insensitive to any vertical force or bending imposed on the test body. Since this gage was fixed to the test stand and could not be rotated, horizontal force in the direction of wave propagation only could be measured.

b. Bending Gage (Shown mounted directly below the horizontal force gage in Figs. 10 and 13, and described in Ref. 1). A four active arm strain gage bridge mounted on a thin walled circular cylinder is the basic configuration of this gage. The strain gages are so arranged that the output of the bridge is four times the output of a single gage. While being insensitive to vertical force, the gage is sensitive to horizontal force applied at a point other than the center of the gage itself.

Although the modified Morrison theory does not predict the axial component of force, it had to be measured in the experimental work in order to determine the vertical or overturning moment. This was necessary because the output of the bending gage was the sum of the actual overturning moment and that due to the horizontal force acting on the body. Overturning moment was found by subtracting the axial force times its moment arm from the output of the bending gate. Because the gage measuring this axial force was fixed in position, overturning moment could be measured only for the orientation of the body axis perpendicular to a wave crest.

c. Vertical Force Gage. (Figs. 16 and 17, and Ref. 1). A Shaevitz linear differential transformer is used as the sensing element in this diaphragm-type instru-

ment. The gage is constructed so that it is sensitive to moment and horizontal force.

d. Torsion Gage (Figs. 12 and 18). No suitable instrumentation was available to measure the lateral or horizontal moment imposed on the body when at an angle of attack with the wave motion. A special gage was therefore designed and constructed to measure this lateral moment.

The basic requirements or specifications of the design were that it be:

i. sensitive enough to measure the torques expected over the range of angles of attack set forth in the experimental program.

ii. insensitive to bending.

iii. insensitive to vertical force.

iv. possess adequate stiffness so as not to adversely lower the natural frequency of the gage assembly.

v. adaptable with regard to mounting to existing gages.

vi. simple and inexpensive to construct.

vii. output suitable for the Sanborn recorder.

Details of the gage are presented in Appendix D and Fig. 18.

e. Wave Gage. (Figs. 19 a,b,c , and Ref. 1). A two wire resistance type wave gage was employed for measurement of the wave profile. Two gages were used, one at the location of the body center and another one wavelength upstream from this point.

f. Recorder. (Fig. 6) A Sanborn Model 150, 4-channel recording oscillograph was the electronic amplifying and recording instrument used in conjunction with all of the gages employed in this report.

g. Support Rod Bracket and Shielding. Considerable time was spent devising a bracket connection between the gages and model support rods that would sustain the forces and moments imposed during the experiments, and at the same time permit the body to be easily changed and adjusted in position.

Figure 13 shows this bracket and the shielding (supported independently of the test stand) that was constructed to eliminate the wave forces on the two support rods.

Two important factors in the design of the shielding were the alignment of the shielding tubes with respect to the support rods and the adjustment of the shielding with depth of the model. The use of large diameter shielding eases the alignment problem but increases the error in measured force and moment due to flow disturbances produced

Figure 6. Wave Tank

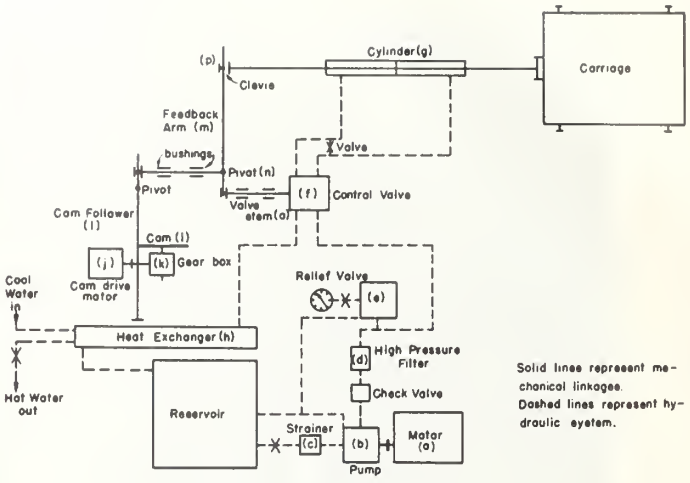


Figure 8. Schematic of wave generator

Figure 7. Wave Generator

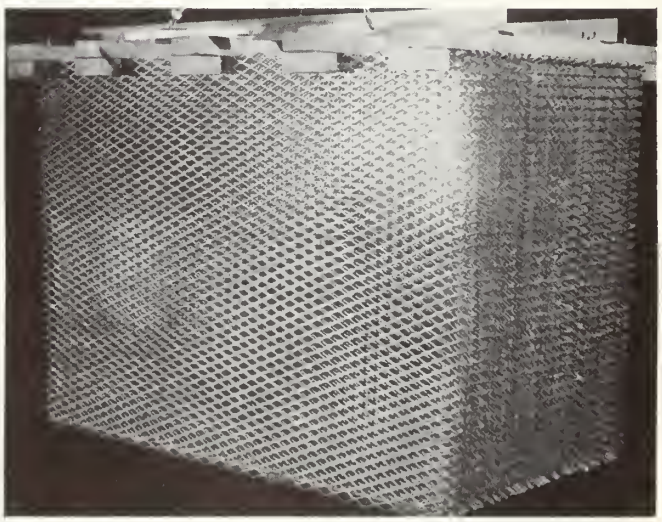


Figure 9 Wave filter

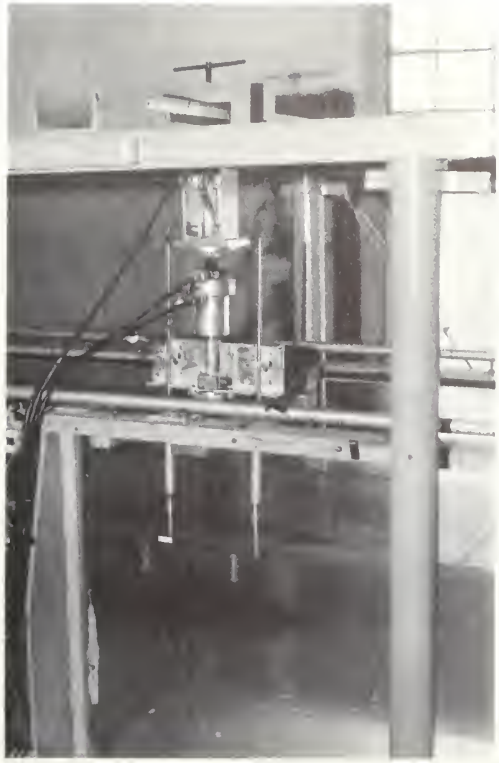


Figure 10 Test Stand

Figure 11 Exp. Models

Figure 12

Torsion Gage

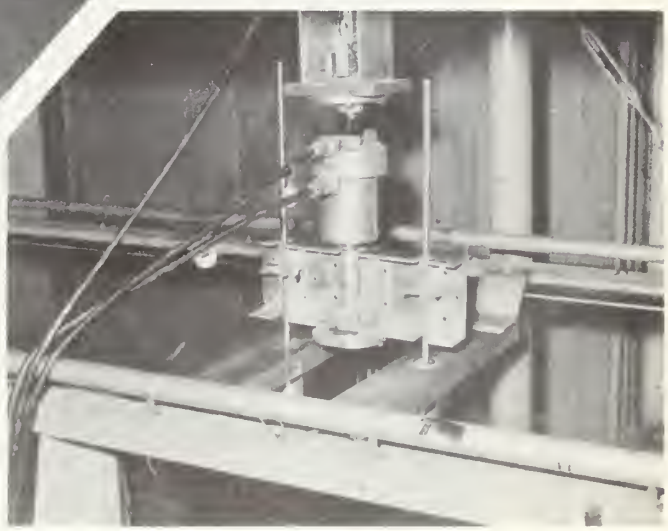


Figure 13

Support Rod Bracket
and Shielding

Figure 14. Horizontal Force Gage

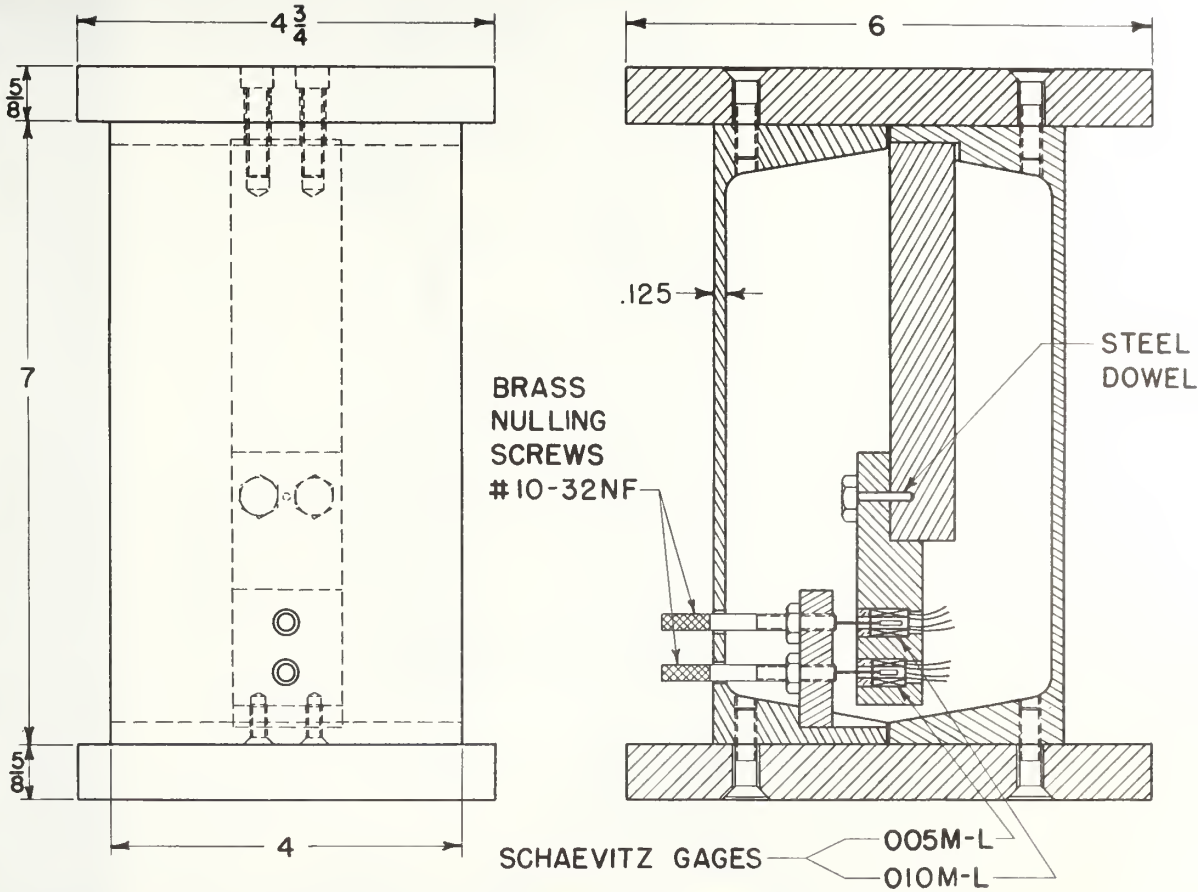


Figure 15. Horizontal Force Gage Schematic

Figure 16. Vertical Force Gage

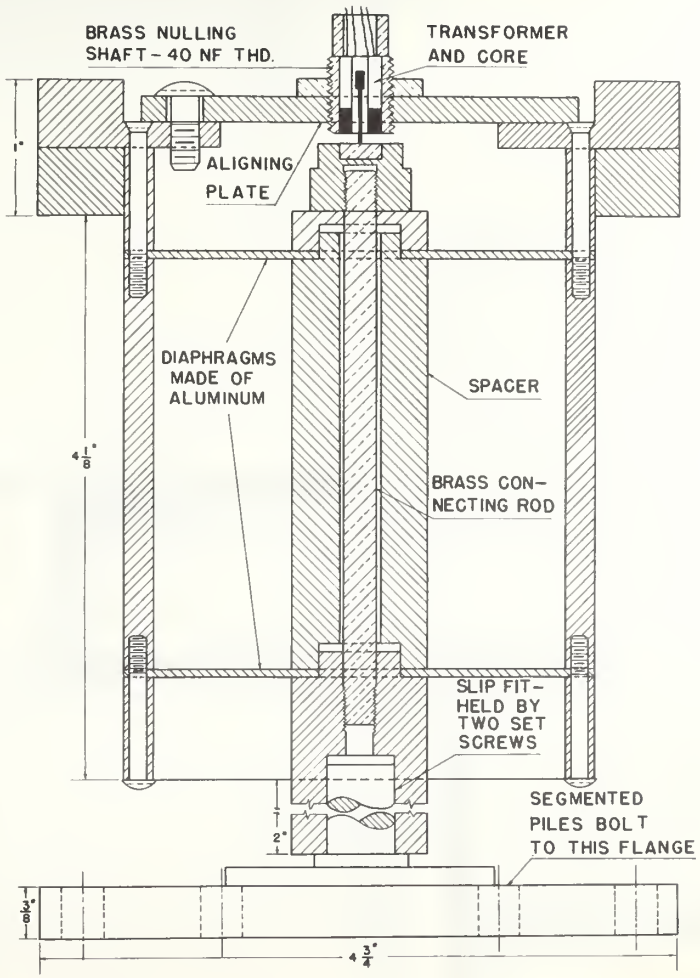


Figure 17. Vertical Force Gage Schematic

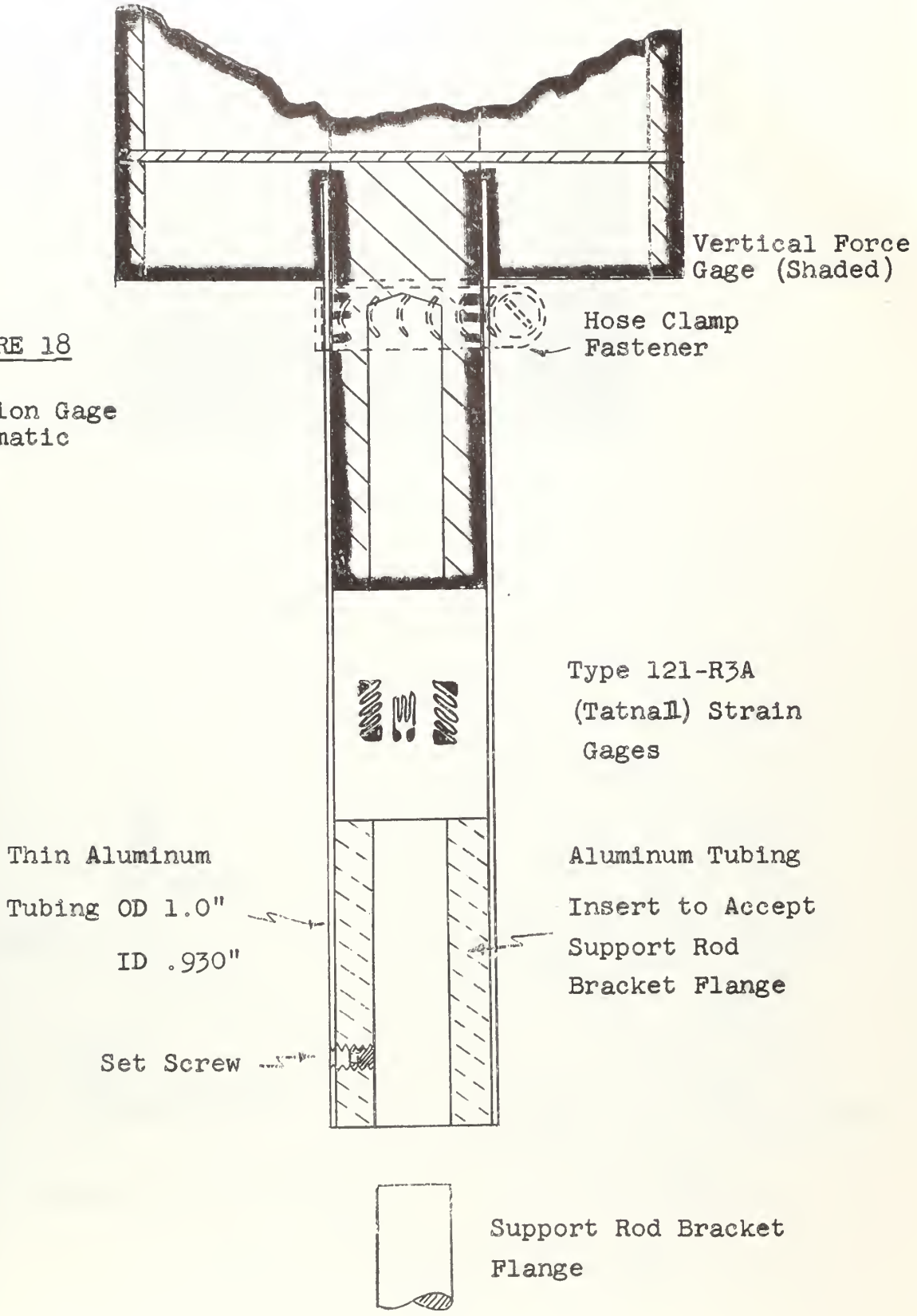
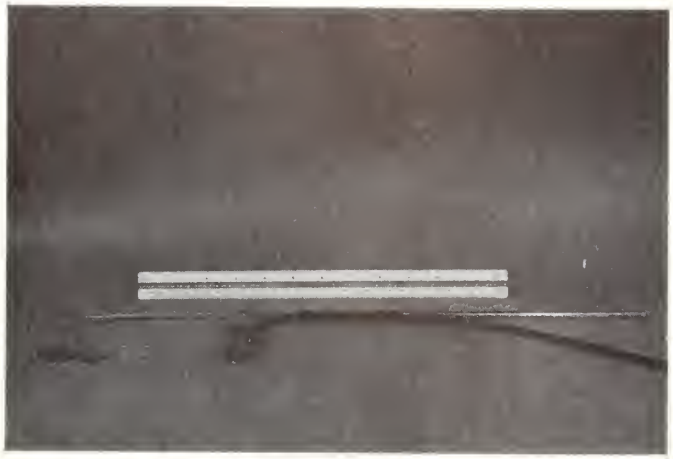
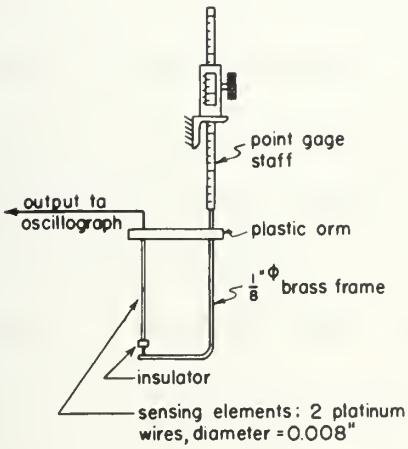


FIGURE 18

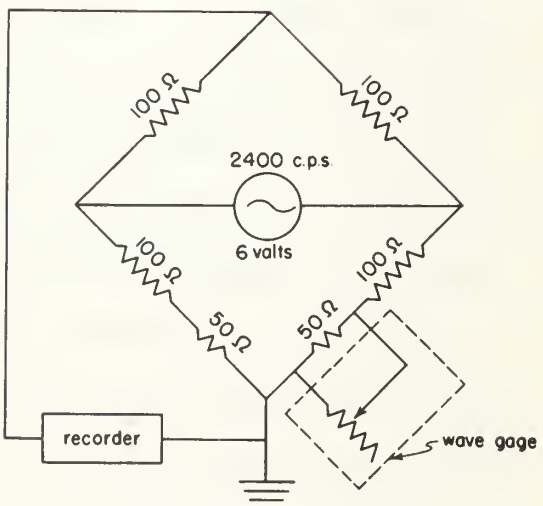
Torsion Gage Schematic



a) Photograph



b) Schematic of Wave Gage



c) Circuit Diagram

Figure 19. Resistance Wave Gage

by the shielding. With this thought in mind, two sets of shielding were constructed (1" O.D. and 1.5" O.D.). The shielding size used was the minimum compatible with the obtained deflection of the support rods.

C. Experimental Procedure

1. Calibration

a. Force and Moment Transducers.

The horizontal force, vertical force, vertical moment, and horizontal moment gages were statically calibrated by applying known forces and recording the response on the Sanborn recorder. The forces were applied so as to obtain the response of the gages in both directions from the neutral position. Since there was a large variation in the buoyancy of the test models, it was known that the zero position of the vertical force gage would be different with each model. Therefore, this gage was calibrated both with no body, i.e., no buoyant force acting, and with the very buoyant four inch cylinder attached. This procedure showed that the response of the gage was linear over the required range. A sample calibration of the horizontal force gage is shown in Fig. 20.

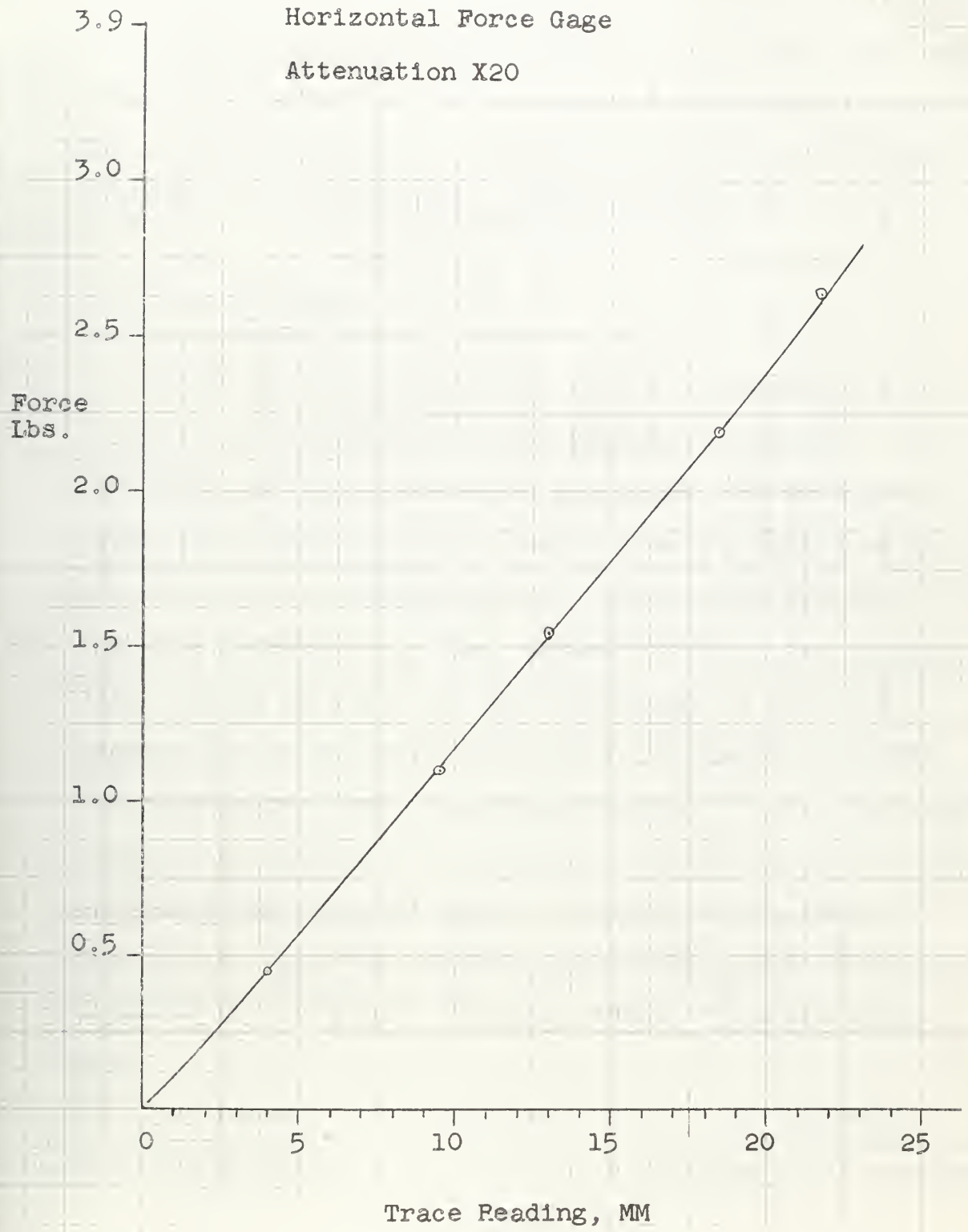


Figure 20. Typical Calibration Curve

RJE

b. Wave Gage.

The resistance-type wave gages were also calibrated "statically" by moving them vertically in equal increments above and below the still water line until near full scale deflection was achieved on the Sanborn recorder. For a discussion of the indirect calibration of the downstream wave gage trace see Section III-C-3-c.

2. Test Procedure

Prior to the commencement of the experimental runs, a complete calibration of each gage was conducted. Upon completion of the calibration procedure, the wave generator was started and the controls were adjusted until the desired wave length and wave height were achieved. The wave generator was then turned off and a few minutes later turned on again without adjusting the controls. Examination of the wave trace from the Sanborn recorder showed that the wave being generated was, within very small limits, the same as the originally established wave. The procedure was repeated several times with the same results; i.e., the wave height, wave length, and frequency remained the same once the wave generator controls were set.

The schedule of experiments as given in Appendix A was then conducted with each test consisting of the passage

of thirty or more waves. In accordance with prior considerations, the tests involved the use of two different waves. The first four runs were made using a wave ten feet in length and approximately seven and one half inches in height (hereafter referred to as the ten-foot wave), and the remaining eleven runs were made using a wave five feet in length and approximately four inches in height (hereafter referred to as the five-foot wave). Upon completion of the test schedule, a complete calibration procedure was again conducted.

3. Data Analysis

a. Data Records

The experimental data was recorded on standard four-channel Sanborn recorder paper. A sample of a typical test record is given in Fig. 21. The test record shown was obtained from a run in which the 2-7/8 inch diameter cylinder was positioned with its longitudinal axis parallel to the direction of wave propagation; i.e., with zero degree angle of attack. In this case, horizontal force, vertical moment, vertical force, and the wave profile were recorded. For those experimental runs in which the model was positioned with an angle of attack of forty-five degrees, the following items were re-

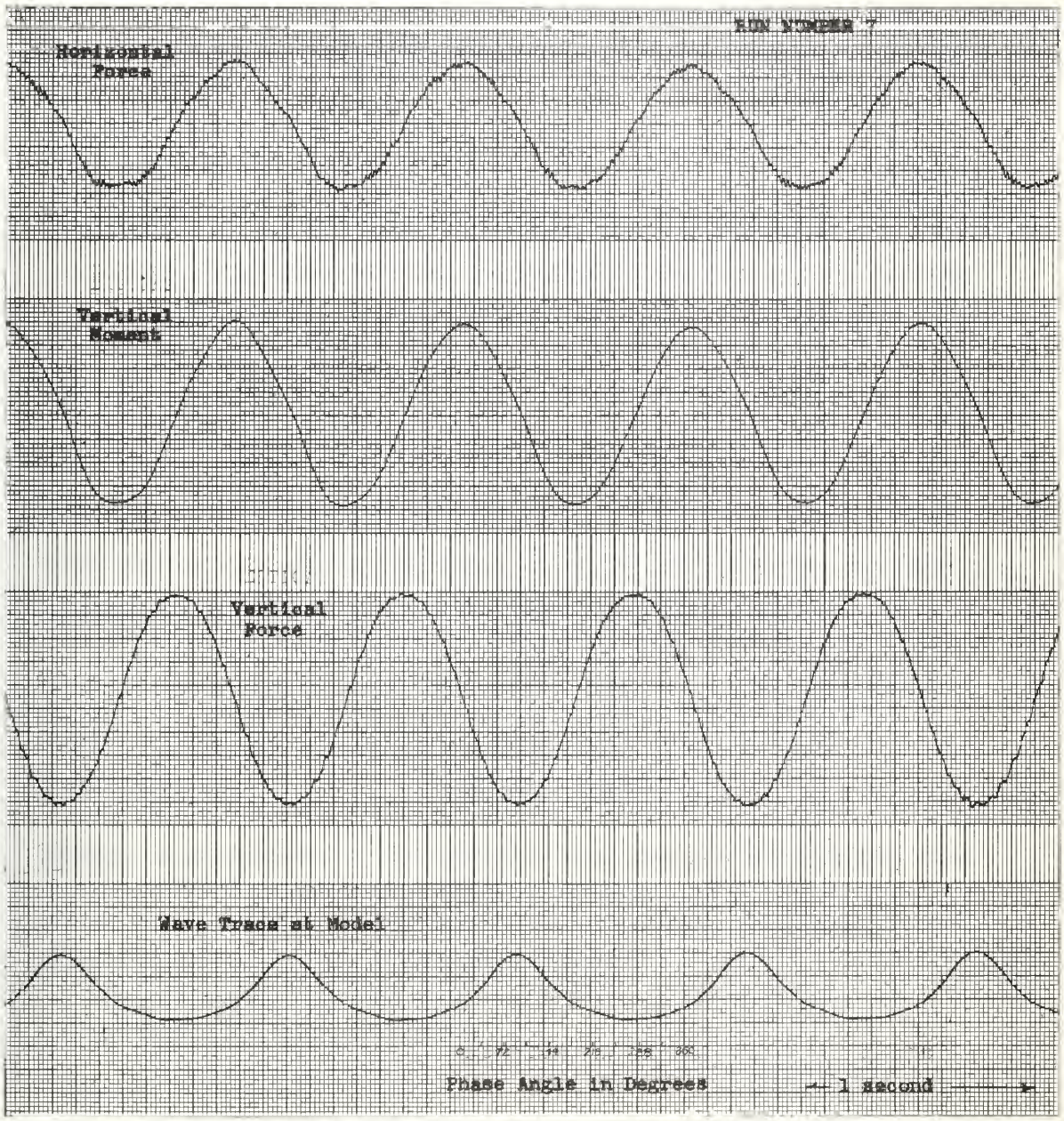


FIGURE 21. SAMPLE TEST RECORD

corded: wave profile from the upstream wave gage, lateral moment, vertical force, and wave profile from the wave gage at the body. These quantities were also measured when the model had an angle of attack of ninety degrees and in addition lateral force was measured.

When the body was oriented with an angle of attack of forty-five degrees, the horizontal force measured by the force gage was not normal or parallel to the body. Therefore one could not obtain the overturning moment directly from the horizontal force and vertical moment gage readings. Because of this and the fact that none of the three theories were used to predict horizontal force acting at an angle with the body other than zero or ninety degrees, neither horizontal force nor vertical moment were recorded on these runs. Vertical moment was not recorded for ninety degree angle of attack runs because with this orientation there should be no overturning moment except in the case of the submarine shape.

From the test records one could obtain the force and moment signatures. To obtain the values of forces and moments at phase angles compatible with those at which these quantities had been predicted by theory, the following procedure was followed.

In accordance with the wave profile equation, the phase angle at the crest was defined as $\psi = 90°$. Since

the wave length was known, it was possible to locate phase angle $\psi = 0°$ at one quarter wave length before the wave crest. Having done this, the wave was divided into ten equal sections and the forces and moments were read at the thirty-six degree intervals for which they had been predicted by the computer program.

It should be noted here that the vertical moment recorded on the test records consists of two components. One component is due to the horizontal force which acted on the model at a known distance from the moment gage. The other component is due to the vertical forces which acted on the body producing what might be called the overturning moment. It is this overturning moment in which we are interested, and thus, the component of vertical moment due to the horizontal force had to be subtracted from the moment recorded on the test record. The accuracy of this subtraction is discussed in Section III-C-3-e.

b. Force and Moment Data Reduction

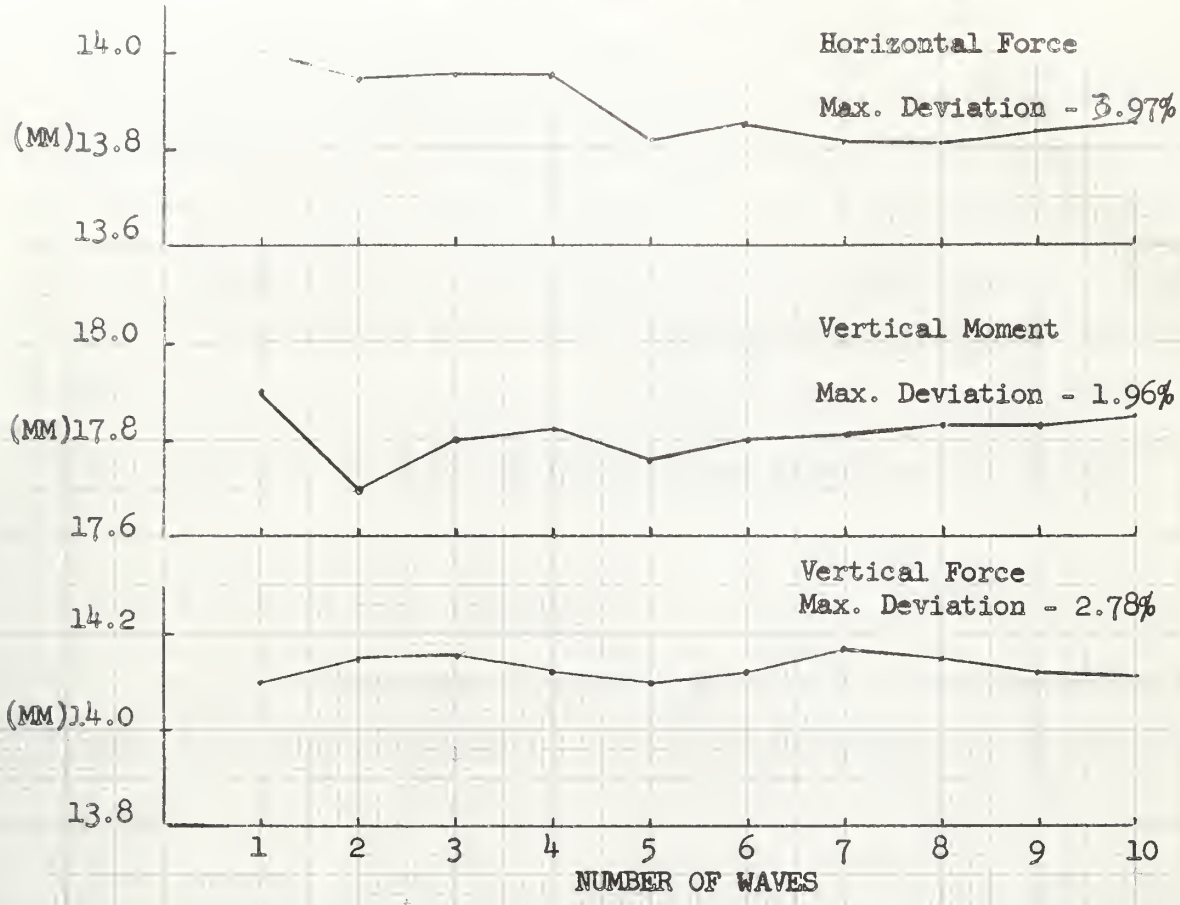
In order to obtain representative force and moment values from the test records, it was at first thought that an averaging process over approximately ten successive waves would be necessary. Therefore, two representative test records were investigated to determine the difference in values which would be obtained by reading

values from one typical wave and from the average of two, three, four, and up to ten successive waves. The progressive mean charts in Fig. 22 are the result of this investigation. The maximum deviation between any one reading and the average of the ten successive readings (found in the horizontal force reading for the five-foot wave) was 3.97 percent. If one carefully selects the wave to be investigated a lesser deviation would be obtained.

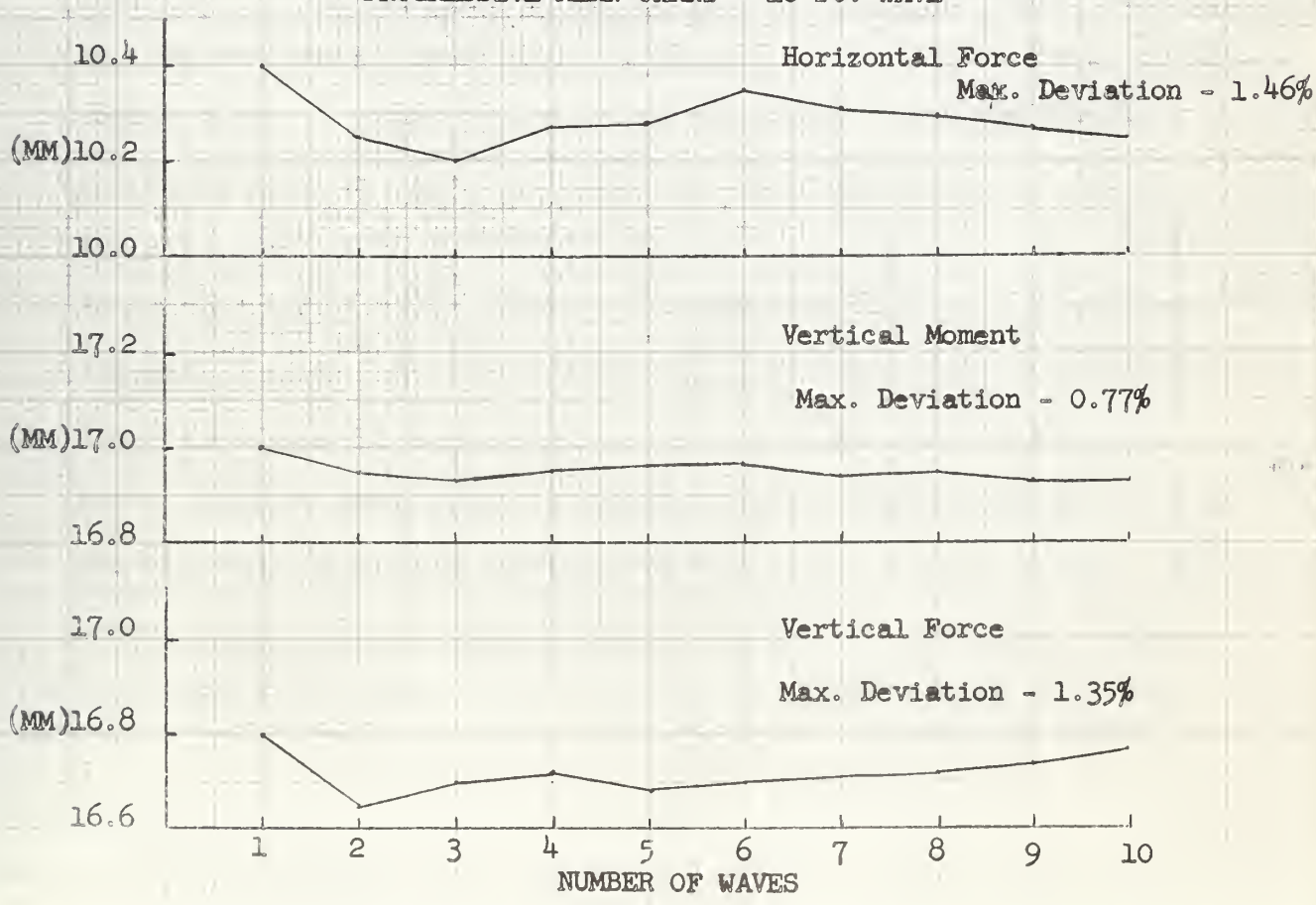
It is felt, therefore, that on the basis of this investigation one would be justified in obtaining the values of forces and moments from one representative wave on each test record. The procedure was followed with care being taken to select a representative wave rather than an unusual one which might differ markedly from succeeding waves.

The data points were read off the test record in millimeters and were converted to pounds force or foot-pounds moment by use of the calibration curves which were obtained as discussed in Section III-C-1-a. Since it was known that the frequency of the forcing function; i.e., the wave, was not greatly different from the natural frequency of the test equipment, an investigation was necessarily conducted to determine whether or not resonance effects were present in the recorded values of forces and moments. For a discussion of this investigation see Section III-C-3-d. The results of the investigation showed that in all cases the phase shift

PROGRESSIVE MEAN CHART - 5 ft. WAVE



PROGRESSIVE MEAN CHART - 10 ft. WAVE



involved was negligible and that there was no significant magnification in the records of vertical force and lateral moment. The values of horizontal force and vertical moment were corrected in accordance with the magnification factors determined in the frequency analysis. Having done this, the experimental results were tabulated in Appendix A and were plotted on the graphs presented in Section IV.

c. Wave Data Reduction

The wave data reduction was complicated by the fact that the wave gage located at the body did not function properly in recording amplitude during the experimental runs. This became evident upon a comparison of the calibration traces made for this gage just prior to the commencement of the experimental runs and immediately upon conclusion of the experiments. These two calibrations differed greatly especially in the region below the still water line. As a result, one could not feel confident in using either calibration trace. Furthermore, neither calibration gave wave heights consistent with those measured visually during the experiments. It is true that it is difficult to measure wave heights precisely by marking the levels reached by the wave crests and wave troughs at the side of the tanks. However, these visual measurements should be accurate within one-half inch. The wave heights obtained by use of the

calibration curves deviated from the measured values by as much as two inches and thus, they were not employed in the wave data reduction.

Some of the test records - in particular, those in which horizontal moment was recorded - contain both the upstream wave gage trace and the downstream wave gage trace. In these cases, it was possible to obtain the wave profile from the upstream wave gage trace and calibration curve. The wave heights obtained from this gage did appear to be consistent with the visually measured heights, the deviation never being greater than one-quarter inch.

Having both wave gage traces on certain records and having a reasonable calibration of the upstream gage, it was decided to attempt to calibrate the downstream wave gage trace on the basis of the upstream trace (assuming of course, no change in wave form between the two gages). To do this, the trace produced by the upstream gage for a given wave was converted from millimeters deflection on the test record to feet by means of the calibration curve. Thus, at any phase angle the amplitude in feet was known from the upstream trace. The deflection in millimeters at corresponding phase angles for the same wave on the downstream wave gage trace were then known to represent a certain number of feet of amplitude. In making this

comparison, one had to be certain he was examining the traces at the upstream and downstream gages which represented the same given wave. Since the wave gages were located one wave length apart, the corresponding wave traces were one length apart on the test record. With this in mind, the back calibration was performed and the resultant calibration curve was plotted.

Examination of this curve showed that the portion above still water was very nearly the same as that obtained in the previous direct calibration attempts. The portion below the still water level was not so well defined since the largest trace deflection available for back calibration was five millimeters. It was noted that the direct calibration curves were nearly parallel from about seven millimeters deflection to twenty-five millimeters deflection and that the slope of these lines was about the same as the slope at the end of the line which was drawn through the back calibration points available. Accordingly, the extension of this curve obtained from back calibration was drawn parallel to the lines from the direct calibration.

Once this back, or indirect, calibration was completed, the amplitudes of the various wave traces were checked. The results of this check showed that the peak amplitudes obtained by using the back calibration curve did correspond closely with those measured visually.

One other check was made on this calibration curve. Noting that the two direct and the one indirect calibration attempts resulted in calibration curves which showed very little variation above the still water line, it was felt reasonable to assume that this portion of the calibration was quite accurate. Furthermore, a wave profile plotted from the upstream wave gage trace showed that the five-foot wave being generated closely approximated a Stokian wave. Therefore, assuming first that the wave crest obtained from the test record was accurate, and second that the wave generated had a Stokian wave profile, the wave height (double amplitude) was calculated on the basis of Stokian wave theory. The wave heights calculated in this manner checked closely with those obtained both from the back calibration curve and by visual measurement. The following table provides for comparison the wave heights obtained by each of these methods for several waves.

Wave Height at Downstream Wave Gage

Wave	Visually	By Back Calibration Using Upstream Gage	From Stokian Theory Using Measured Crest Heigh
1	7.5 inches	7.4 inches	7.6 inches
2	7.5 inches	7.3 inches	7.3 inches
3	4.0 inches	4.0 inches	3.9 inches

On the basis of the preceding discussion, it was decided to obtain the wave profiles using the back calibration curve. These profiles are plotted with the results in Section IV and are tabulated in Appendix A. Although one could not say that the wave profiles obtained in this manner are precise, it is felt that they are reasonably accurate and that the peak values are accurate to within 0.2 inches.

d. Frequency Analysis

From the theory of mechanical vibrations, it is well known that the response of a mechanical system to a periodic forcing function will be subject to increasing magnification and phase shift as the frequency of the forcing function approaches the natural frequency of the mechanical system. When working with wave forces then, it is desirable to insure that the natural frequency of the assembled test equipment is at least seven to ten times the frequency of the waves being considered (1) in order to avoid the resonance effect. If this condition is not met, the resonance effects; i.e., magnification factor and phase shift, must be determined for each run.

The assembled test equipment, including model and support rods, was analyzed as a single degree of freedom

system subject to a periodic force and to some unknown damping force. Examination of the damped natural frequency record for the system used revealed that the damping force did not satisfy the logarithmic decrement condition of linear damping. The damping of bodies oscillating in a fluid is proportional to the hydrodynamic drag. At moderate Reynolds number, the steady state drag coefficient is constant and the drag is therefore proportional to the velocity squared. Although it has been shown that the steady state drag coefficient does not apply exactly to oscillating systems (4), it is reasonable to assume that the damping force is proportional to the velocity squared.

For square law damping the equation of motion may be written as:

$$m\ddot{x} + C_2\dot{x}^2 - kx = F_o \sin \omega t \qquad (37)$$

in which the symbols are defined as follows:

m is the mass of the system;
C_2 is the damping coefficient for square law damping;
k is the spring constant for the system;
F_o is the maximum value of forcing function;
ω is the frequency of the forcing function.

Jacobsen (12) has obtained an approximate solution of this non-linear differential equation by substituting an equivalent linear coefficient of damping. The equation then becomes

$$m\ddot{x} + C^{*}\dot{x} + kx = F_{o} \sin \omega t \tag{38}$$

in which C^{*} is the equivalent linear coefficient of damping.

The approximate equivalence of these two systems is established by letting the work done per cycle by the linear damping force be equal to the work done per cycle by the square law damping force. Therefore the following equation may be written.

$$\Delta W = 4 \int_{o}^{\frac{\pi}{2\omega}} C_{2} \dot{x}^{2} \, dx = 4 \int_{o}^{\frac{\pi}{2\omega}} C^{*}\dot{x} \, dx \tag{39}$$

Noting that C_{2} and C^{*} are constant over one cycle and assuming the response of the system is periodic and of the form:

$$X = X_{o} \sin (\omega t - \theta) \tag{40}$$

equation (39) may be rewritten as follows:

$$C_{2}x^{3}\omega^{3} \int_{o}^{\frac{\pi}{2\omega}} \cos^{3}(\omega t-\theta)dt = C^{*}x^{2}\omega^{2} \int_{o}^{\frac{\pi}{2\omega}} \cos^{2}(\omega t-\theta)dt$$

Since the values of the integrals do not depend upon θ, one may perform the integration and solve for C.

$$C^* = \frac{8}{3\pi} \times \omega C_2 \tag{41}$$

This expression may be used in obtaining the value of the ratio of the equivalent linear damping to critical damping. If this ratio is known, one may enter a standard plot of magnification factor or phase shift versus forced to natural frequency for linearly damped systems and thereby determine the magnification factor and phase shift. Considering both the damped free vibration and damped forced vibration, the following expression can be written:

$$\frac{C^*_{forced}}{C^*_{free}} = \frac{\omega x_{forced}}{\omega x_{free}} \tag{42}$$

$$C^*_{forced} = \mathcal{J}^*_{forced} \cdot C_c \; ; \quad C^*_{free} = \mathcal{J}^*_{free} \cdot C_c \tag{43}$$

Substituting for C^*_{forced} and C^*_{free} from equation (43) and rewriting equation (42) results in the following:

$$\mathcal{J}^*_{forced} = \frac{\omega x_{forced} \mathcal{J}^*_{free}}{\omega_d x_{free}} \tag{44}$$

where ω_d is the damped natural frequency of the system. In this equation ω and x_{forced} can be read from the test

records, and ω_d , x_{free}, and $\mathcal{J}^*_{free}$ can be determined from the free vibration record. In the case of linear damping, the logarithmic decrement can be written as follows:

$$\ln \frac{x_A}{x_B} = 2\pi \mathcal{J}^*_{free} \Big/ \sqrt{1 - (\mathcal{J}^*_{free})^2} \qquad (45)$$

where x_A and x_B are the amplitudes of any two successive vibrations.

Therefore, by selecting any two successive vibrations, $\mathcal{J}^*_{free}$ can be determined for that cycle. If that value of $\mathcal{J}^*_{free}$ and the value of x_A are substituted into equation (44), $\mathcal{J}^*_{forced}$ can be determined.

The plots available in the literature are made with the abscissa equal to $\frac{\omega}{\omega_n}$, the ratio of the frequency of the forcing function to the undamped natural frequency of the system. From the test records the ratio $\frac{\omega}{\omega_d}$ is available. However, the following expression holds:

$$\frac{\omega_d}{\omega_n} = \sqrt{1 - (\mathcal{J}^*_{free})^2}$$

From this it is clear that $\frac{\omega}{\omega_n}$ is very nearly equal to $\frac{\omega}{\omega_d}$ for small values of $\mathcal{J}^*_{free}$. For each of the experiments conducted, the value of $\mathcal{J}^*_{free}$ was less than 0.04 and, therefore, in these cases $\frac{\omega}{\omega_d}$ may be considered equal to $\frac{\omega}{\omega_n}$.

In accordance with the preceding, the values of magnification factor and phase shift were determined for each test and were applied to the experimental results during the data reduction process. A table of magnification factors and phase shifts found is given below.

TABLE II. MAGNIFICATION FACTOR AND PHASE SHIFT

Body	Wave	Magnification Factor	Phase Shift
4 m. cylinder	10	1.03	0
4 m. cylinder	5	1.06	0
2-7/8 cylinder	10	1.05	0
2-7/8 cylinder	5	1.10	0
1 in. cylinder	10	1.00	0
1 in. cylinder	5	1.00	0
Submarine shape	10	1.03	0
Submarine shape	5	1.05	0

e. Error Estimate

A complete error analysis has not been attempted. However, because of the subtraction procedure involved in obtaining the value of overturning moment, it was considered essential that an estimate of the maximum error possible in this quantity be made. As discussed in

Section III-C-3-b, the overturning moment was evaluated by subtracting the component of vertical moment due to horizontal force from the total vertical moment measured by the bending gage. Of the two components of vertical moment, the component due to horizontal force was by far the greater. Therefore, the procedure entailed the subtraction of a factor of nearly the same magnitude as the total measured vertical moment from the total measured vertical moment. This resulted in a relatively small factor for overturning moment. From this it is clear that a small percent error in total moment, or in moment due to horizontal force, would result in a large error in overturning moment. This effect is most severe when the overturning moment is the least percent of the total vertical moment. In particular, the overturning moments determined on the experiments using the ten-foot wave would be subject to the largest errors.

An estimate of the maximum error was made using the following simplified method:

$$M_Z = M_V - F_X \cdot d$$

$$\ell n\, M_Z = \ell n(M_V - F_X \cdot d)$$

$$\frac{d(M_Z)}{M_Z} = \frac{d(M_V - F_X \cdot d)}{M_V - F_X \cdot d} \qquad (45)$$

in which the terms are defined as follows:

M_z is the overturning moment.

M_v is the total vertical moment. (measured at gage)

F_x is the horizontal force acting on the body.

d is the moment arm; i.e., the distance from the center of the test body to the center of the bending gage.

Since we are looking for the maximum error, we are interested in the maximum value of $\frac{d(M_v - F_x \cdot d)}{M_v - F_x \cdot d}$. This will obviously be infinite when overturning moment is zero. For non-zero overturning moments we will examine the maximum value of the numerator $d(M_v - F_x \cdot d)$. To estimate this term, the maximum error in reading M_z was assumed to be 1/4 millimeter, which in the case when the attenuation X10 scale was used, is equivalent to ± 0.05 ft.lbs. It was assumed that the moment arm could be measured exactly and that the maximum error in reading F_x was 1/4 millimeter, which in the case of the attenuation X20 scale is equivalent to $\pm$ 0.06 lbs. Since the moment arm was measured to be 33.5 inches, the maximum error in this component of total moment is $\pm$ 0.167 ft. lbs. The maximum error in overturning moment then is estimated to be:

$$\frac{d(M_z)}{M_z} = \frac{0.050 + 0.167}{M_v - F_x \cdot d}$$

Specifically for test run number 1, the estimated error in peak value of overturning moment is:

$$\frac{d(M_z)}{M_z} = \frac{0.217}{0.446} = 48.6\ \%$$

This simplified analysis was made to provide an estimate of the error which might obtain in the evaluation of overturning moments by the method used in these experiments. The accuracy of the gages and recording instruments has not been considered but rather only the accuracies involved in reading the recorded data have been considered. However, this does show the sensitivity of the resulting overturning moment to any inaccuracy in the reading of the Sanborn recorder trace. It should be emphasized that this applies only to the overturning moment and not to the other quantities reported. Furthermore, it should be noted that sensitivity results from the method used to obtain overturning moment and that this method was dictated by the limitations of the instrumentation.

IV. PRESENTATION AND DISCUSSION OF RESULTS

A. Presentation of Results

1. Graphical Presentation

The theoretical and experimental results are presented in graphical form in this chapter as an aid to the reader in making a qualitative comparison of the results. Each graph presents a complete time signature of the results obtained from the modified Morrison theory, from the Stokian potential theory, and from the experimental runs. The following points concerning the graphs should be noted:

(1) Theoretical axial force is plotted for the Stokian potential flow only since the modified Morrison theory had not been used to predict this quantity.

(2) The sine wave potential theory results are plotted for run number 4 only.

(3) For the runs with non-zero angles of attack, experimental results are not plotted for each of the quantities predicted by theory. This is because limitations in the instrumentation prevented the experimental measurement of axial force, horizontal force, and overturning moment for the 45° angle of attack runs, and vertical moment in the 90° angle of attack runs.

2. Tabular Presentation

The complete theoretical and experimental results are presented in tabular form in Appendix A. The tables of theoretical results are presented directly as they were received from the IBM 1620 computer. The input data, identified by the computer program symbols, is printed at the top of the page. The output quantities are printed in eleven groups corresponding to phase angles from 0° to 360° in 36° increments. The quantities within each group are identified in the format statement included with each table.

B. Discussion of Results

1. Model Results - Theory and Experiment

Within the accuracy of the experiments, the measured results show general agreement with the results predicted by the modified Morrison formulation developed in this report. In all cases the prediction of phase angle seems very good and with the exception of overturning moment, in which the experimental values are subject to large errors as discussed in Section III-C-3, the quantitative values of forces and moments as computed by the modified Morrison formulation show fair agreement with the experimental values. Furthermore, if one assumes that the force distribution on the body predicted by theory is as accurate as the magnitude of the

total vertical force predicted, then one may conclude that the overturning moments predicted by the modified Morrison theory are of similar accuracy.

Although the potential theory and modified Morrison theory are completely independent and are based on widely differing assumptions, their results show striking agreement for all cases except those in which viscous forces become important. One would expect this difference in the theoretical results of the two theories in the case where viscous effects are relatively large since the potential flow theory does not account for viscous effects. This point is illustrated in the curves for run number 4 which shows a difference in both magnitude and phase angle predicted by the two theories. Furthermore, this curve indicates that the modified Morrison theory produces closer agreement with experimental results. A rough idea of the relative importance of viscous and inertia forces for each run is given by the parameter $\frac{u_{max}T}{D}$. This parameter can be shown to be proportional to the ratio of the maximum values of viscous to inertia forces.

Irregularities are observed for all runs with the ten-foot wave (Runs 1-4) at phase angles of about 270°. These irregularities may be attributed to the presence of a significant second harmonic in the profile (Stokian) of this wave. This second harmonic produces differences in the values of accelerations and velocities encountered in the

wave crests and troughs. The results for the five-foot wave, which has a negligible second harmonic, do not exhibit this tendency. This point is illustrated by the plot of the moment curve obtained from the sine wave potential theory included for run number 4. This curve does not have this "hump" since there are no higher harmonics and thus the velocities and accelerations encountered in the crests and troughs are of equal magnitude.

The submarine model experiments were conducted only for angles of attack of 0°, 45°, and 90°; however, the theoretical forces and moments are included in the tables and are plotted on the graphs for the additional cases of 180°, 235° and 270° to show the effect of body geometry. (This was accomplished analytically by simply changing the sign of the coefficients for the odd powers of x in the fourth degree polynomial describing the body shape).

It was noted at the time the experiments were conducted that the torsional gage was not, in fact, completely independent of bending for all orientations (α) of the gage. For the runs with 45° angle of attack, however, it was possible to orient the gage such that bending effects were eliminated. For the 90° angle of attack runs, it was not possible to accomplish this because of an obstruction between the torsion gage and the mounting bracket. Thus,

for the submarine shape at 90° (run number 14), bending effects are present in the lateral moment values obtained experimentally and they must be interpreted with this in mind. For runs at $\alpha = 0$, of course, the horizontal moment is not present.

2. Prototype Results - Theory Only

For the purpose of comparing the theories in the prototype range, the forces and moments imposed on a 250 foot long, 32 foot diameter submarine and on a 3 foot diameter, 30 foot long cylindrical structural member have been calculated for a severe sea state. The wave selected (400 ft. long, 25 ft. high) corresponds to a sea wave generated by a 38 knot wind (13). The exact input data and results are included in Appendix B. The results of both theories show close agreement in both magnitude and phase angle for the submarine prototype. In the case of the prototype structural member, the theoretical results differ markedly in both phase angle and magnitude. This is to be expected from the model results since this configuration corresponds closely to that of the 1 inch cylinder in the 10 foot wave. An examination of the modified Morrison theory indicates that the viscous forces are relatively large for this case ($\frac{u_{max}T}{D} = 11.85$).

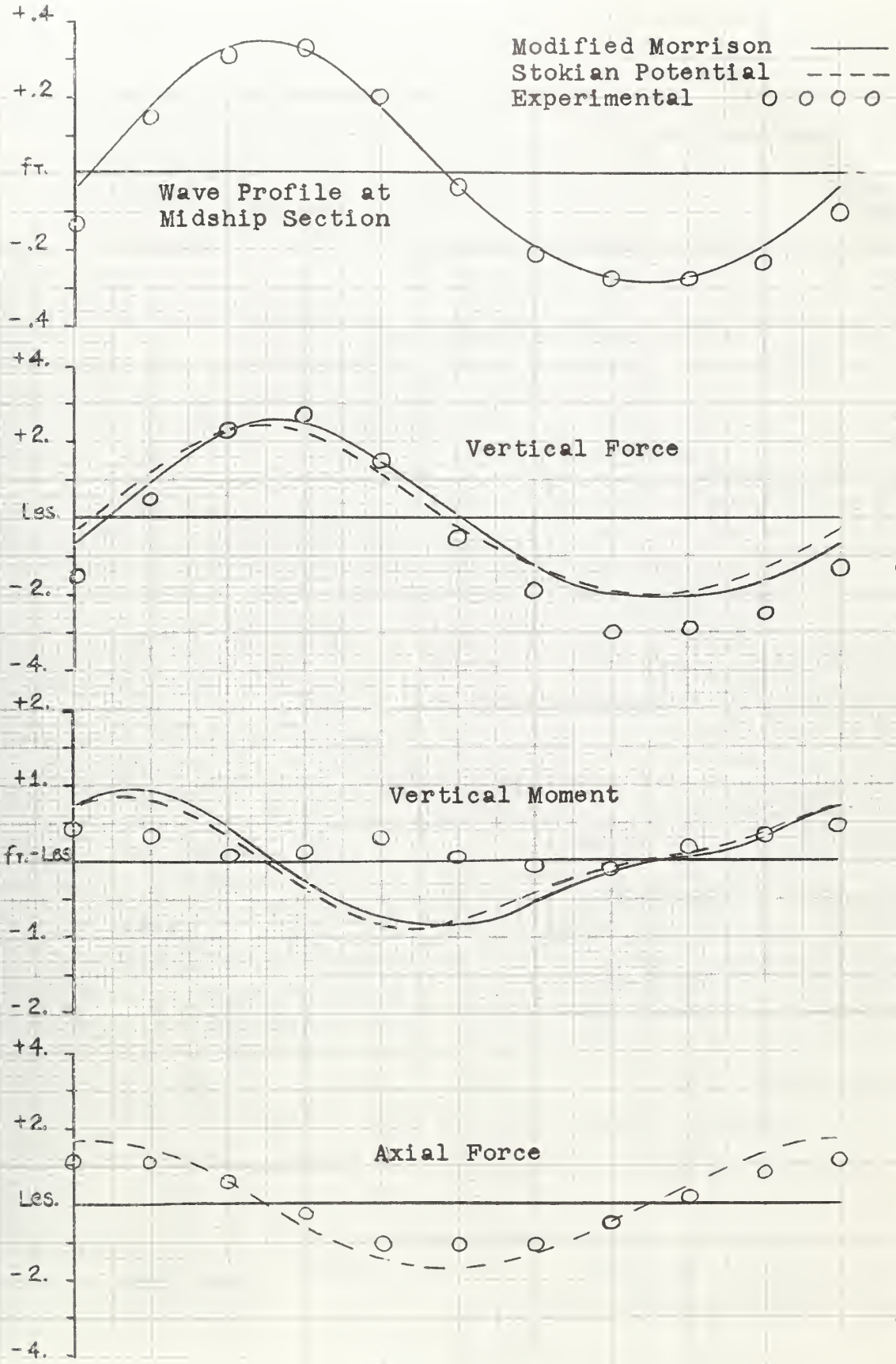
It is noted that for the prototype structural members and for the 1 inch cylinder model (run number 4) the potential theory predicted larger forces and moments than did the modified Morrison formulation. This was unexpected since the fact that the potential theory neglects the large viscous effects would lead one to believe that the potential theory would result in values of lesser magnitude. This difference may be explained by considering the value of C_m used in the modified Morrison theory. In these particular cases, the average C_m obtained from Fig. 4 and used in the modified Morrison theory is in the range of 0.8. However, the "C_m" implicit in the potential flow solution for a circular cylinder is 2.0 in all cases. With the exception of the 1 inch cylinder (run number 4), the C_m determined from Fig. 4 is in the range of 2.0 and close agreement between the potential theory and the modified Morrison theory is observed. Thus, the difference in the theoretical results is accounted for by two effects:

(1) The increased magnitude of viscous force accounted for in the modified Morrison theory but neglected in the potential flow theory.

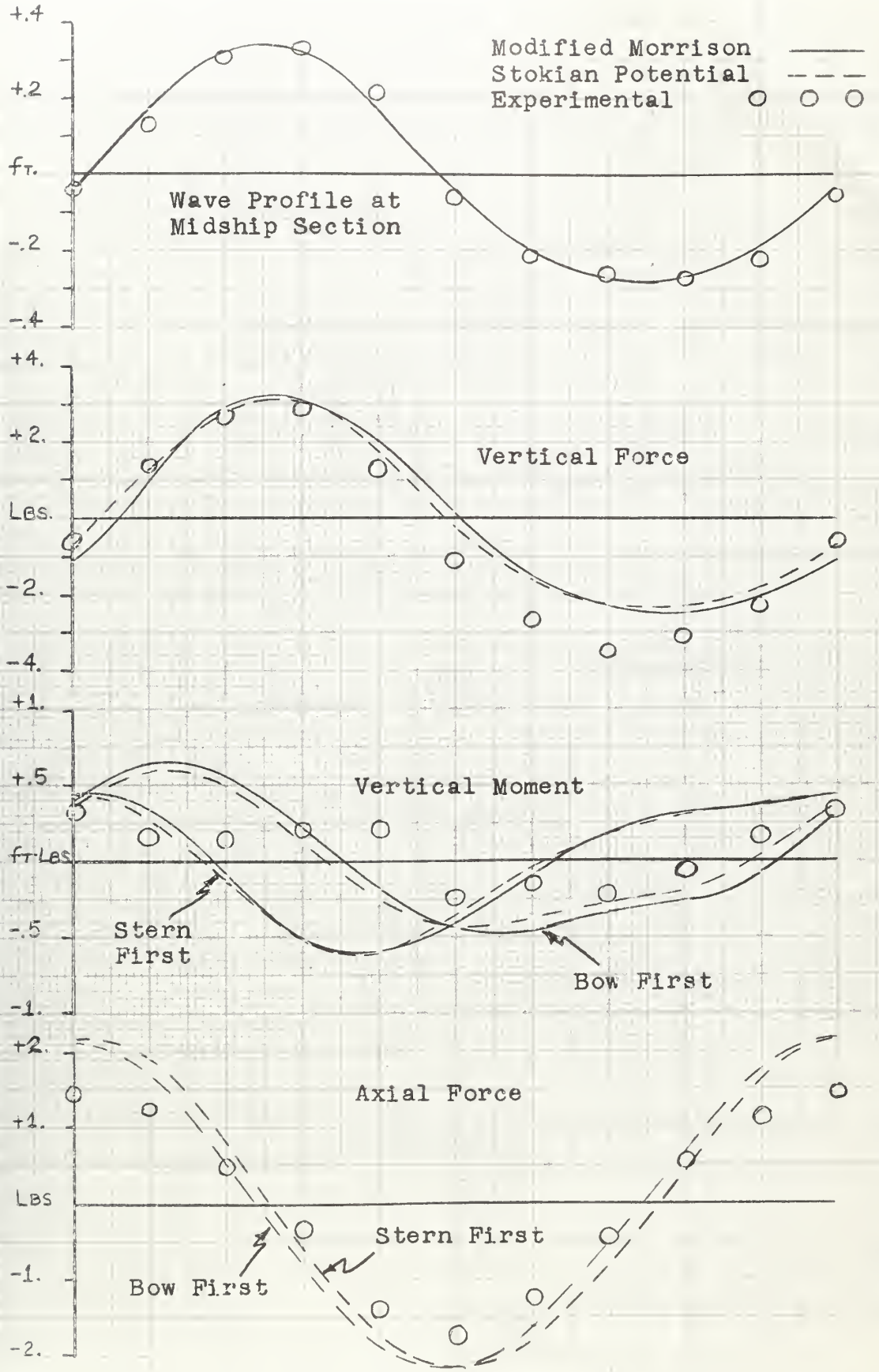
(2) The decreased magnitude of inertial force due to the reduced C_m employed in the modified Morrison theory as compared to constant value of 2.0 implicit in the potential theory. This

difference is present only where viscous forces are large and likely indicates the previously discussed phenomena of flow separation on the body shape effective in inertia calculations.

It should be noted at this point that the modified Morrison theory did in fact show closer agreement to the experimental results in this range.


Modified Morrison
Stokian Potential
Experimental
+.4
+.2
ft.
-.2
-.4
Wave Profile at
Midship Section
+4.
+2.
Les.
-2.
-4.
Vertical Force
+2.
+1.
ft.-Les.
-1.
-2.
Vertical Moment
+4.
+2.
Les.
-2.
-4.
Axial Force
4" cylinder, 10 ft. wavelength, angle of attack 0°, axis 10" below still water
Run number 1

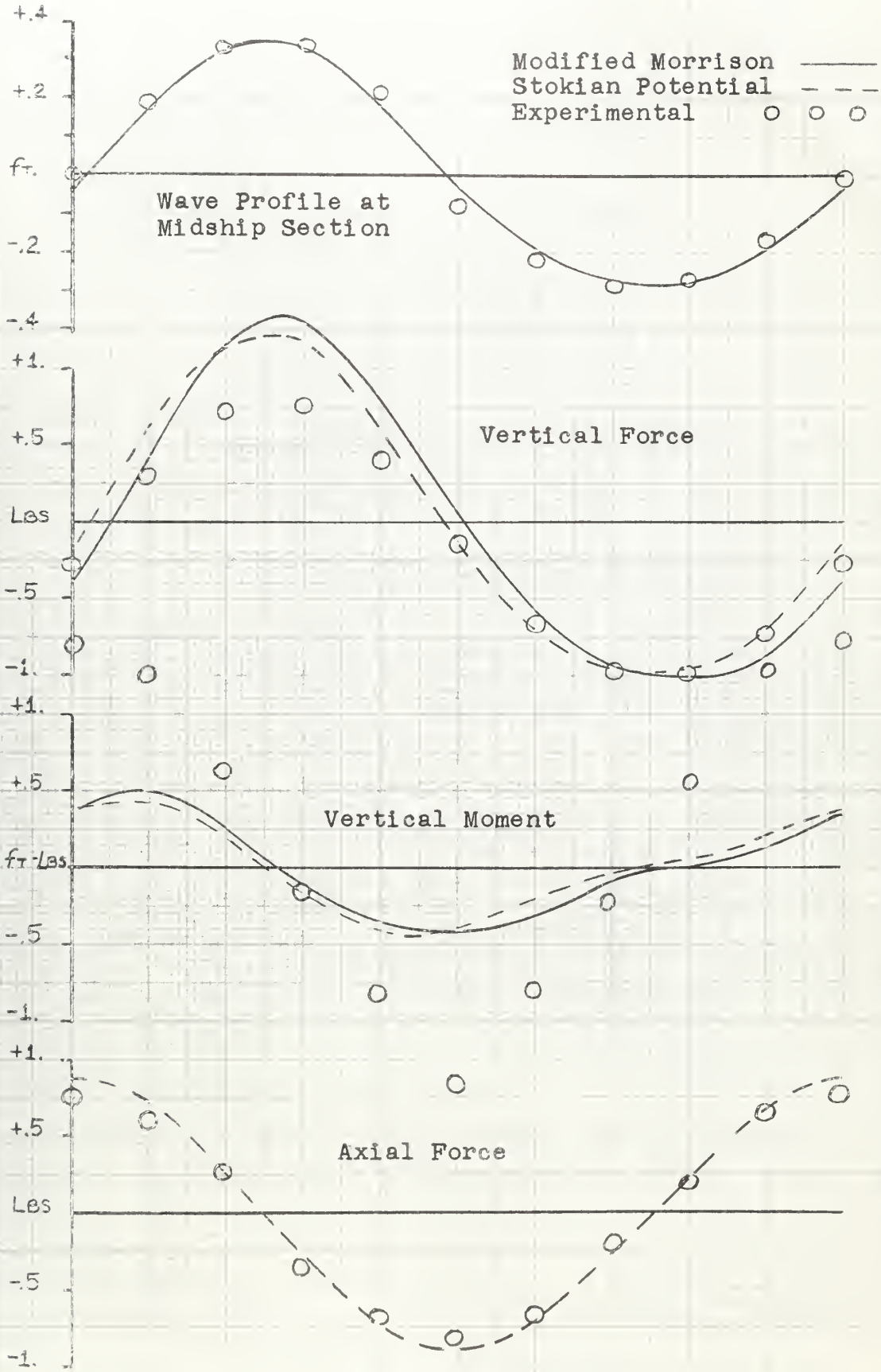
RJE



Submarine shape, 10 ft. wavelength, angle of attack 0°, axis 10" below still water

Run number 2

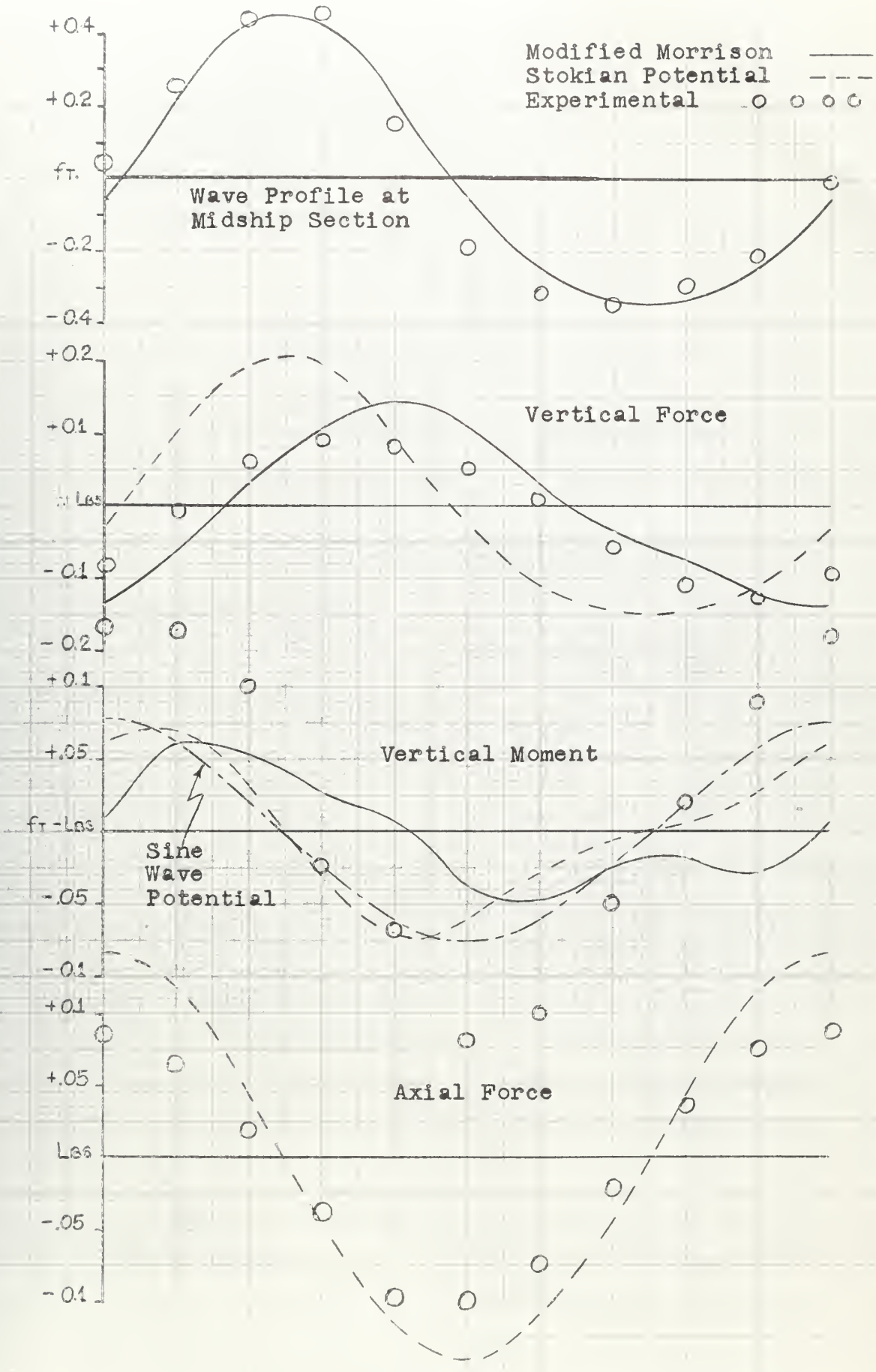
RJE



2 7/8" cylinder, 10 ft. wavelength, angle of attack 0°, axis 10" below still water

Run number 3

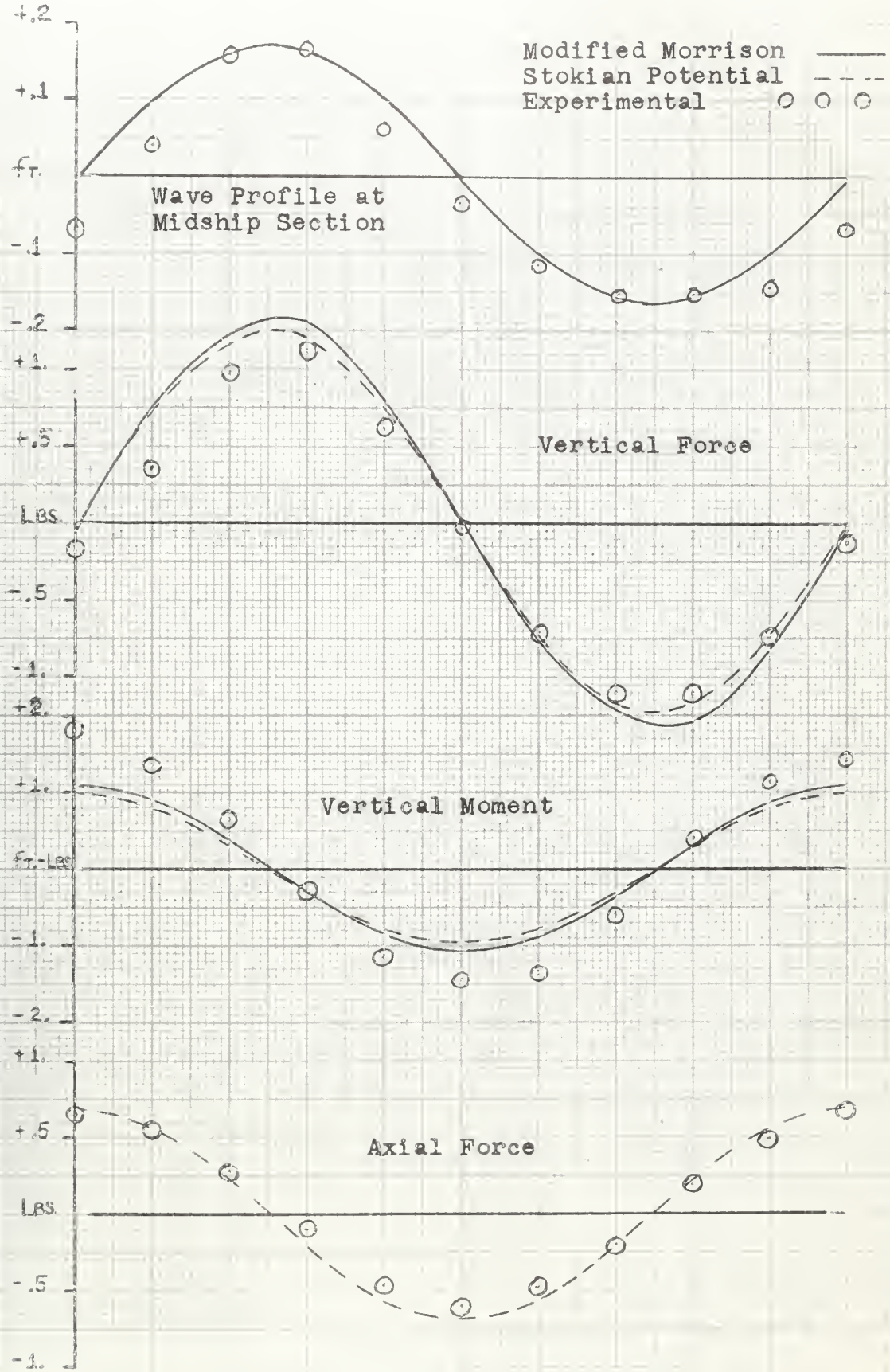
RJE



1" cylinder, 10 ft. wavelength, angle of attack 0°, axis 10" below still water

Run number 4

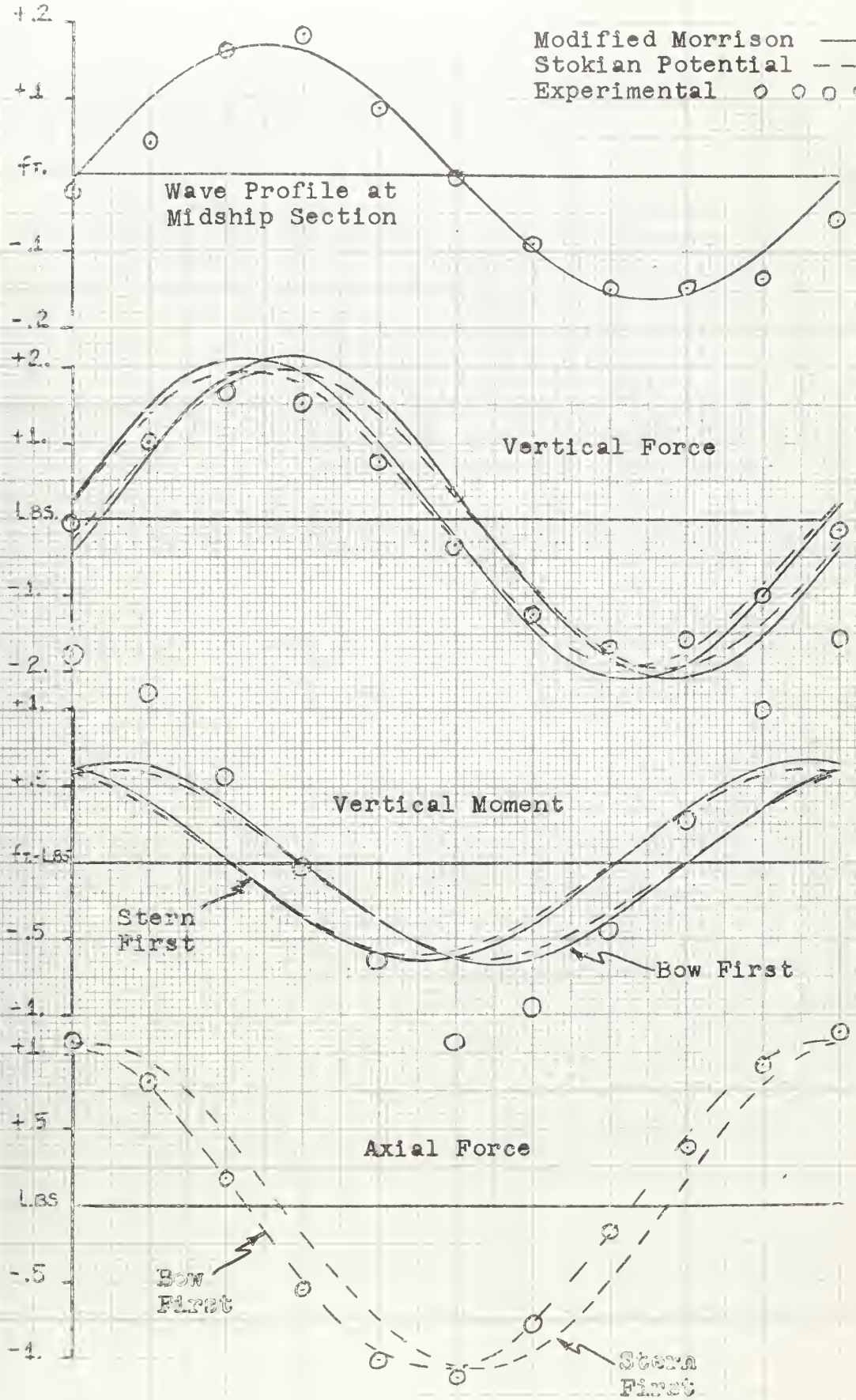
RJE



4" cylinder, 5 ft. wavelength, angle of attack 0°, axis 10" below still water

Run number 5

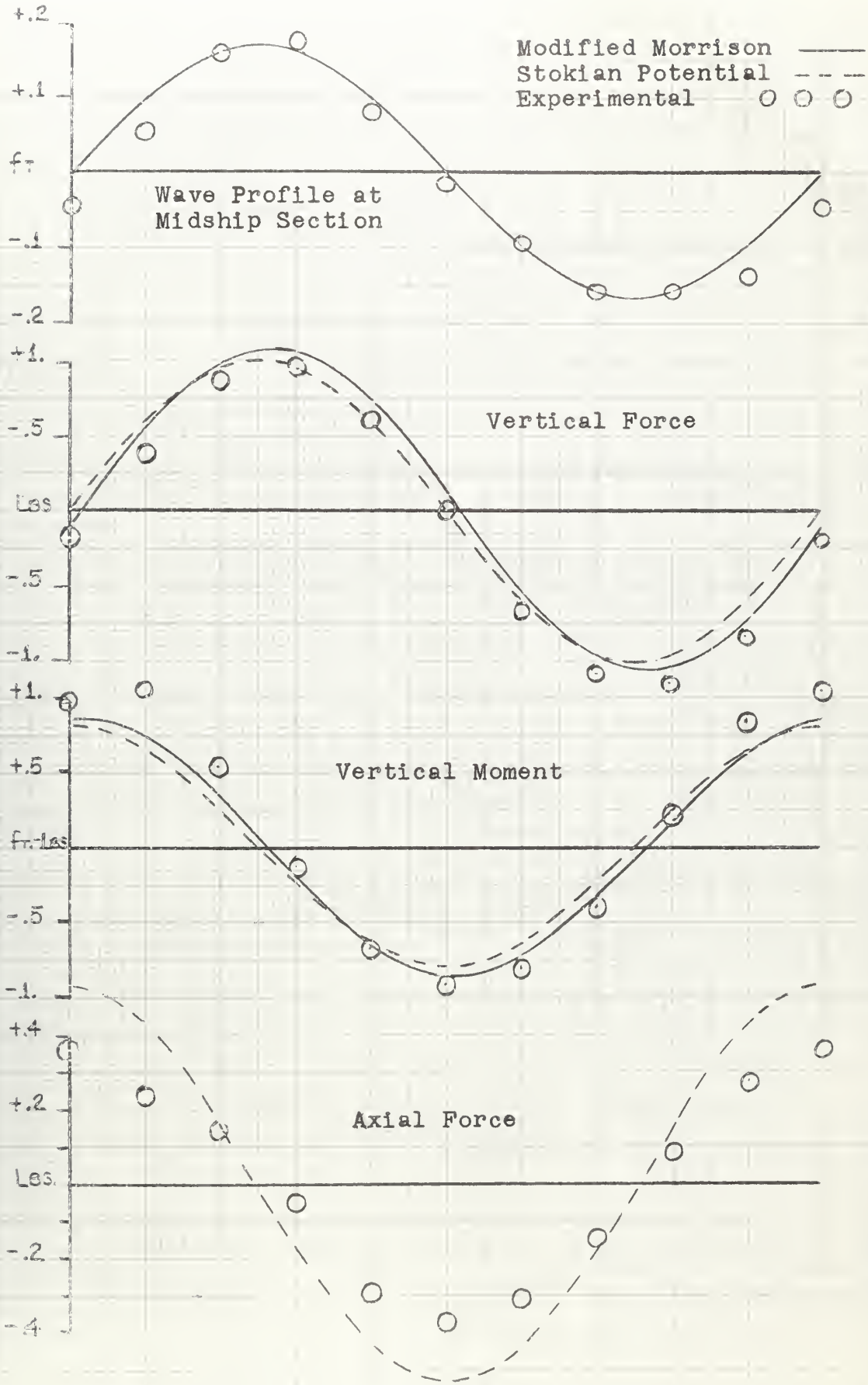
RJE



Submarine shape, 5 ft. wavelength, angle of attack 0°, axis 10" below still water

Run number 6

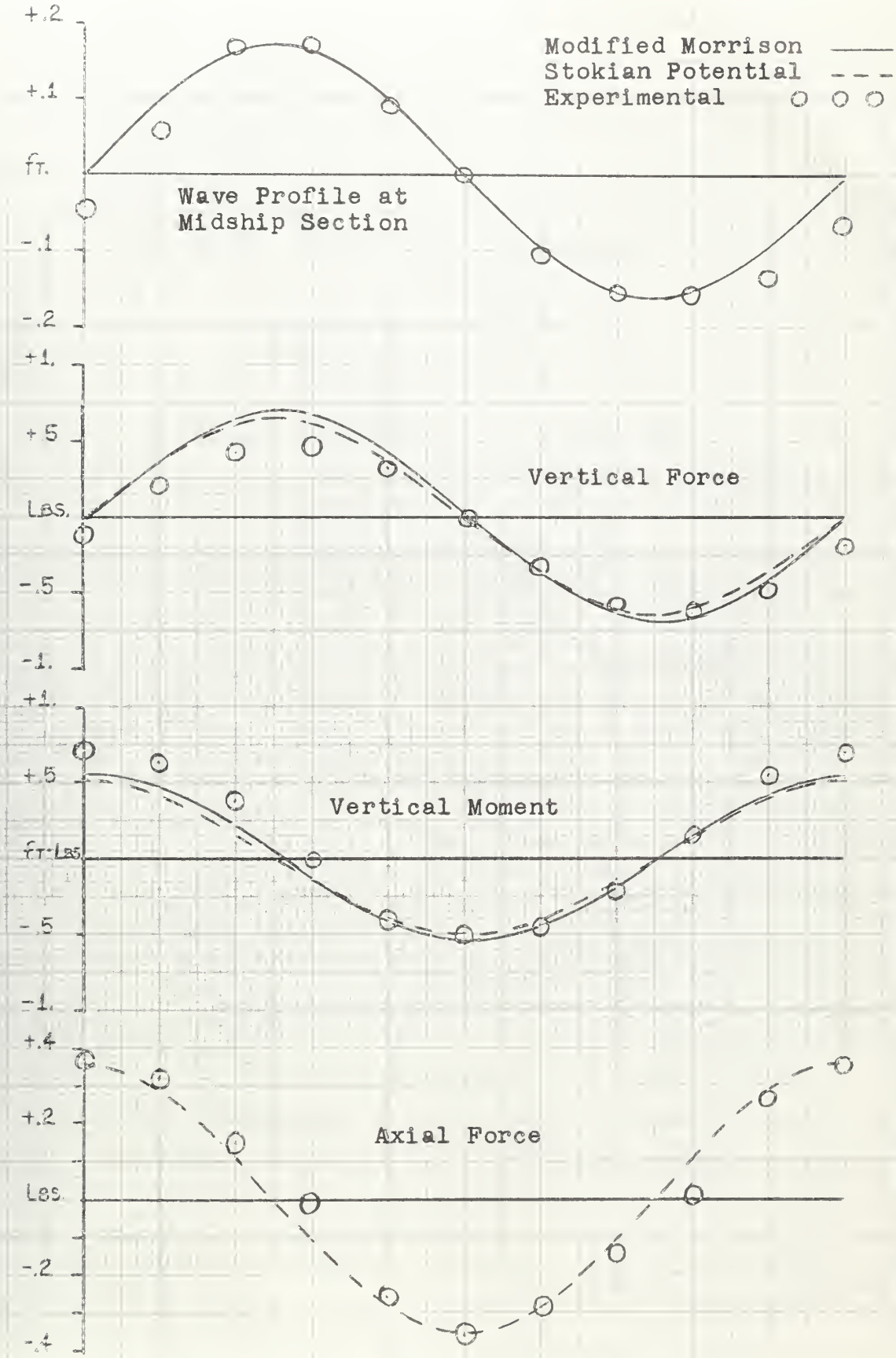
RJE



2 7/8" cylinder, 5 ft. wavelength, angle of attack 0°, axis 6" below still water

Run number 7

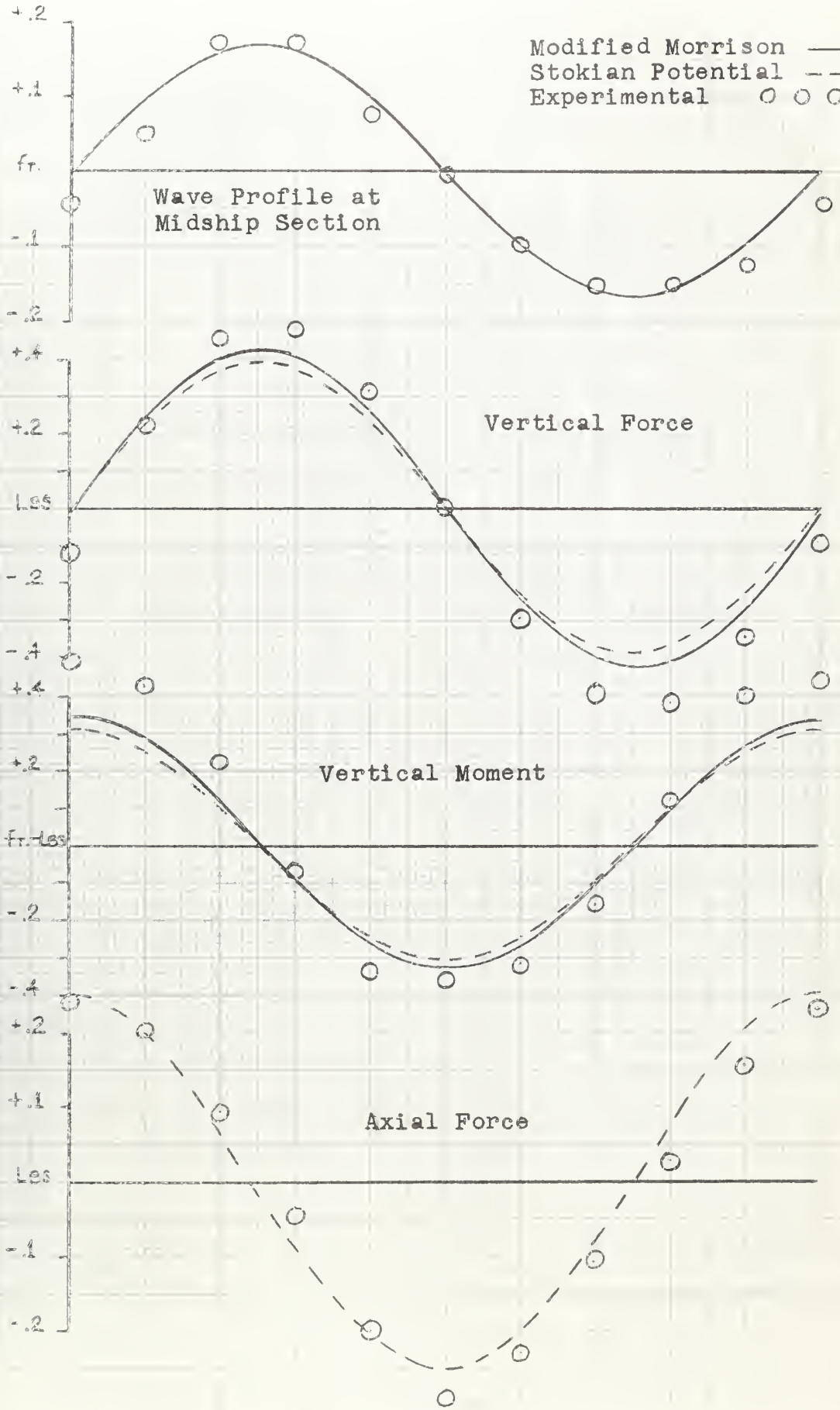
RJE



2 7/8" cylinder, 5 ft. wavelength, angle of attack 0°, axis 10" below still water

Run number 8

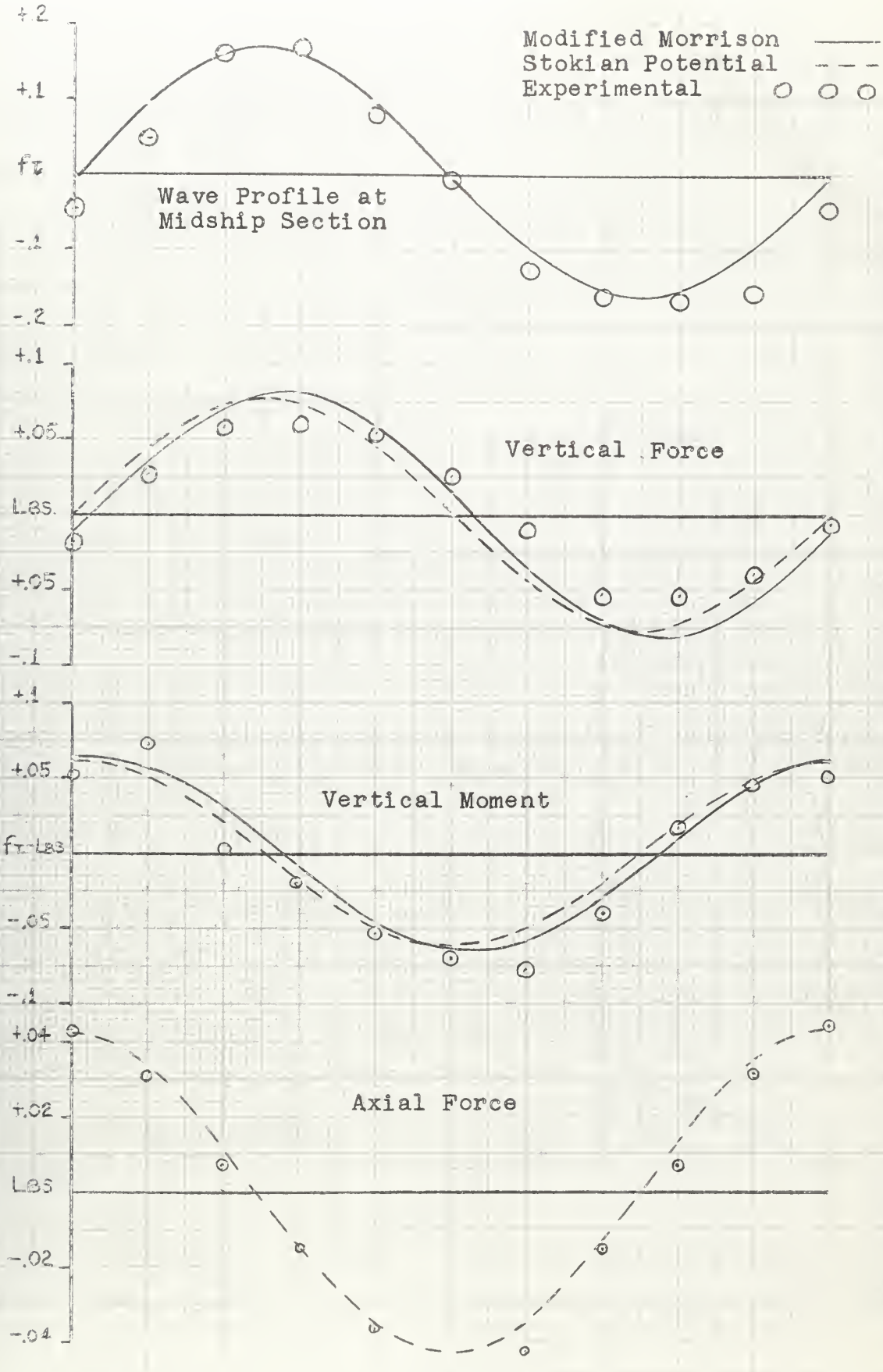
RJC



2 7/8" cylinder, 5 ft. wavelength, angle of attack 0°, axis 14" below still water

Run number 9

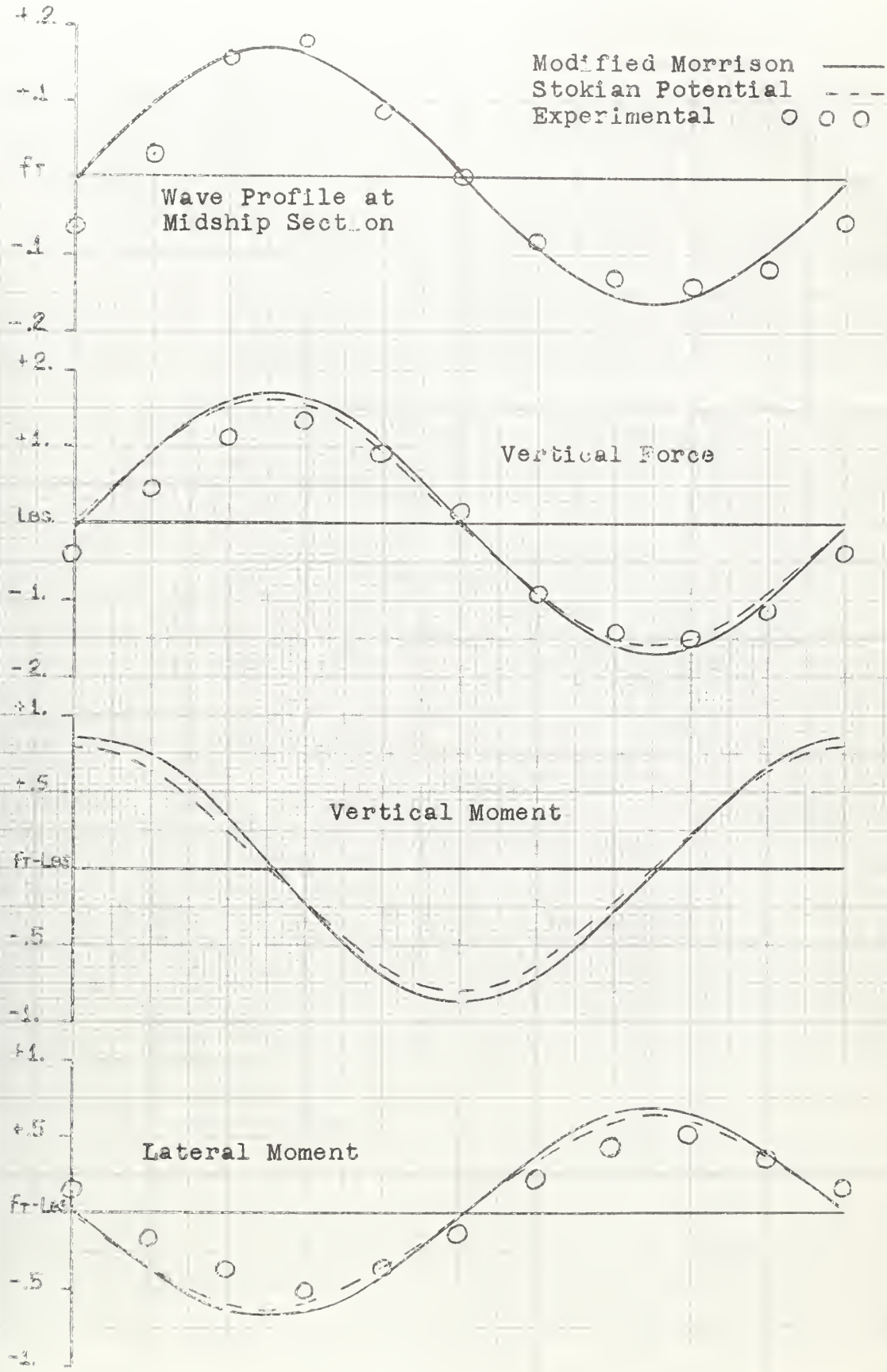
RJE



1" cylinder, 5 ft. wavelength, angle of attack 0°, axis 10" below still water

Run number 10

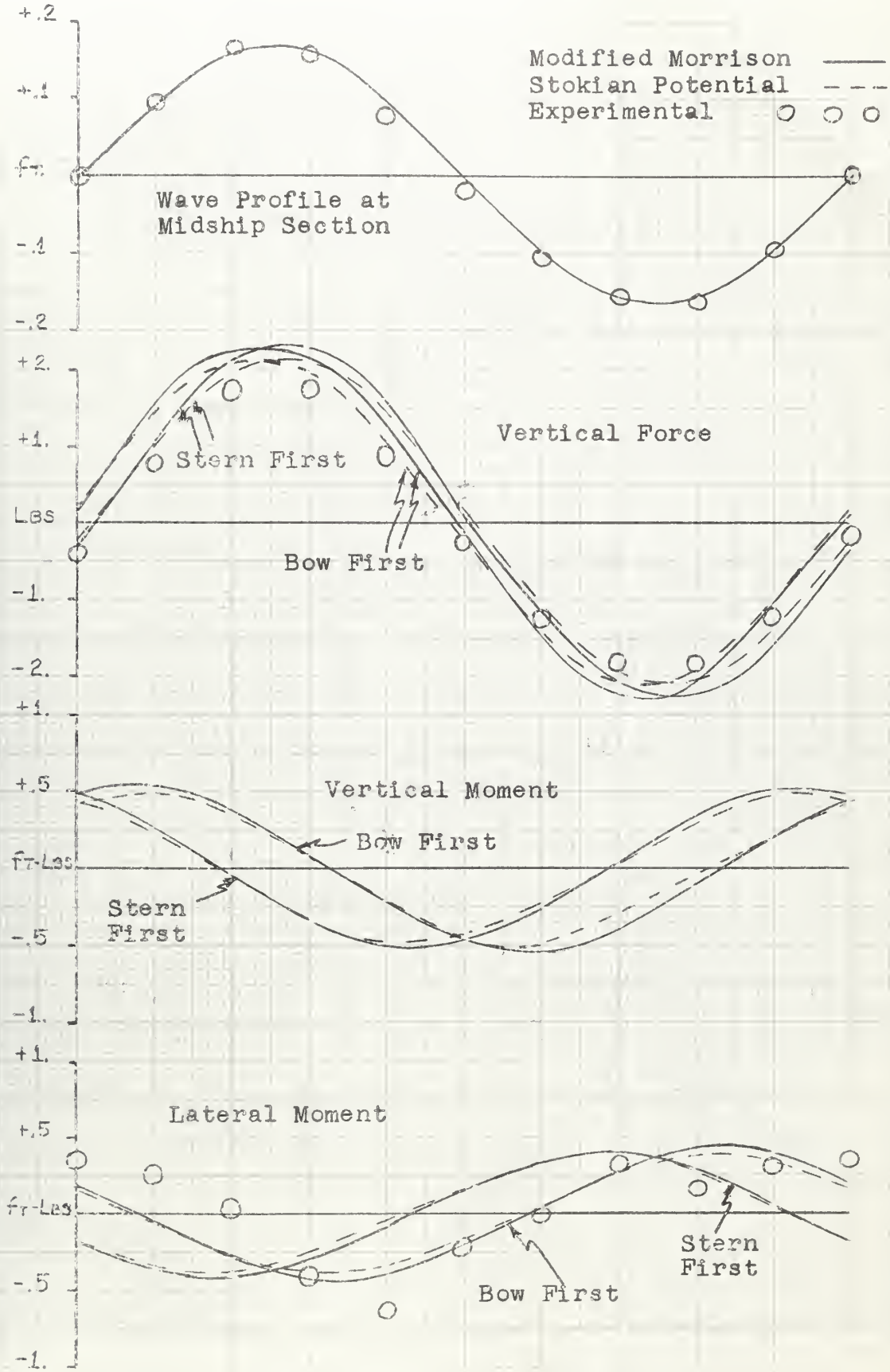
RJE



4" cylinder, 5 ft. wavelength, angle of attack 45°, axis 10" below still water

Run number 11

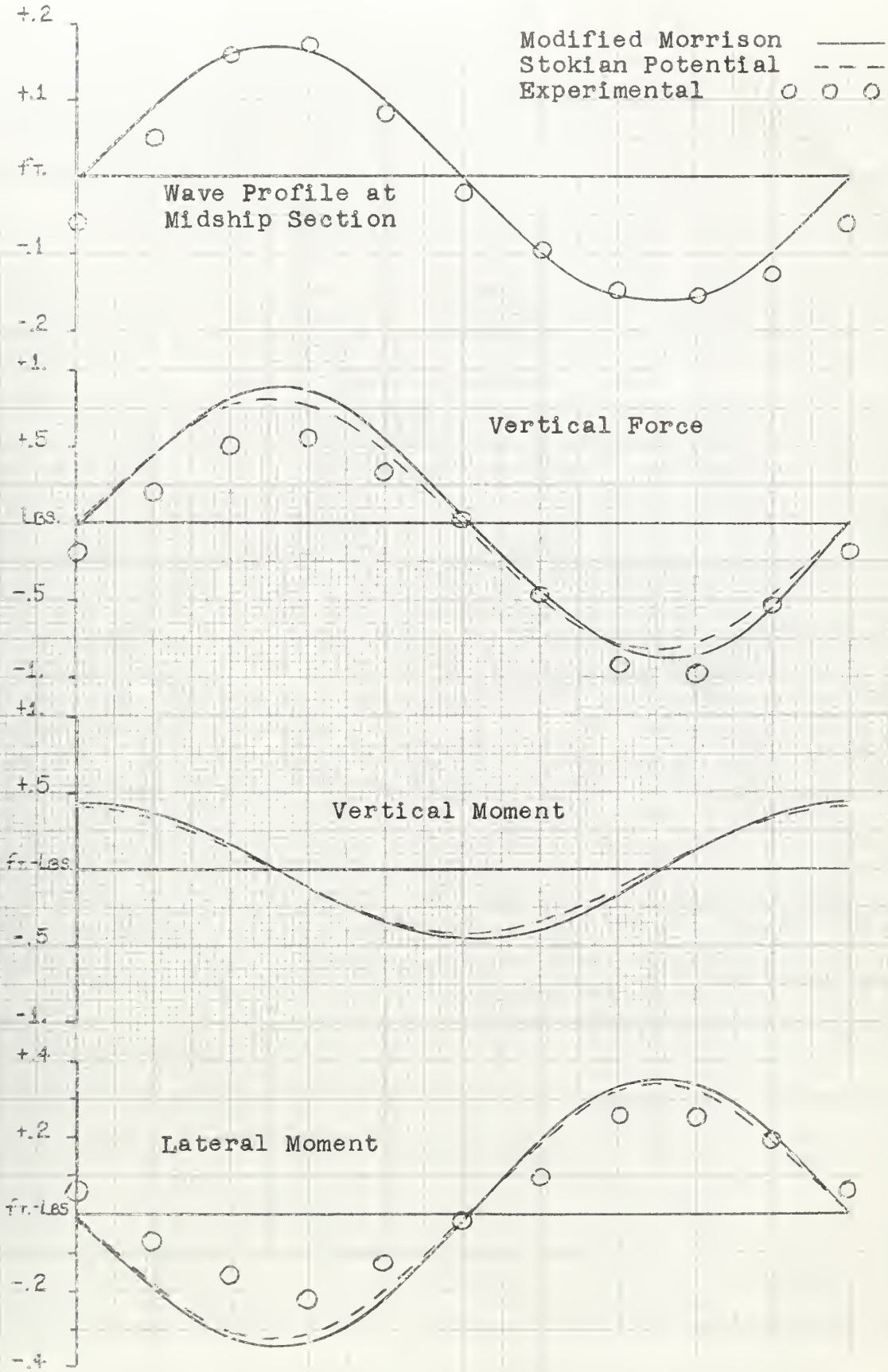
RJE



Submarine shape, 5 ft. wavelength, angle of attack 45°, axis 10" below still water

Run number 12

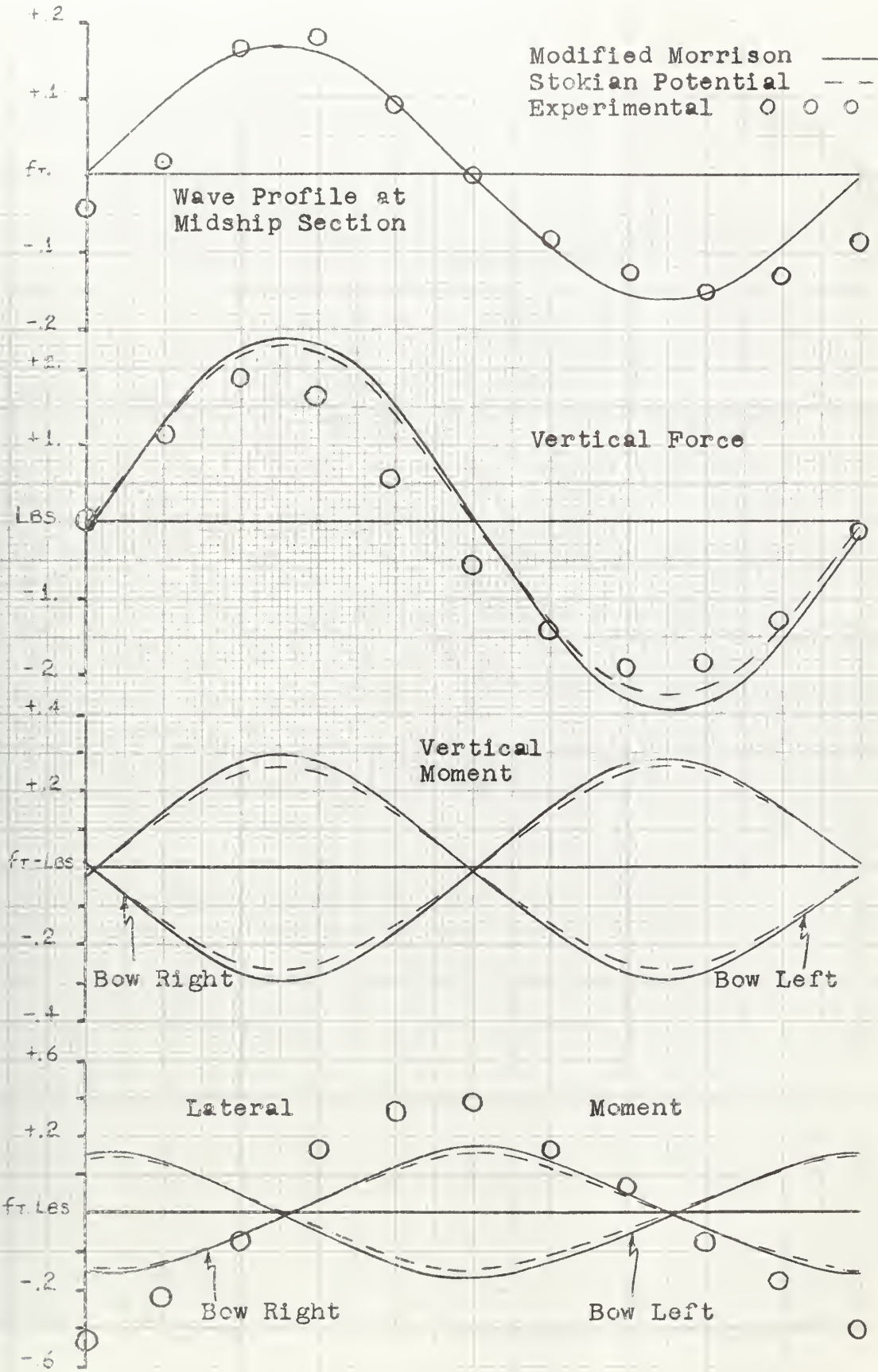
RJC



2 7/8" cylinder, 5 ft. wavelength, angle of attack 45°, axis 10" below still water

Run number 13

RJE

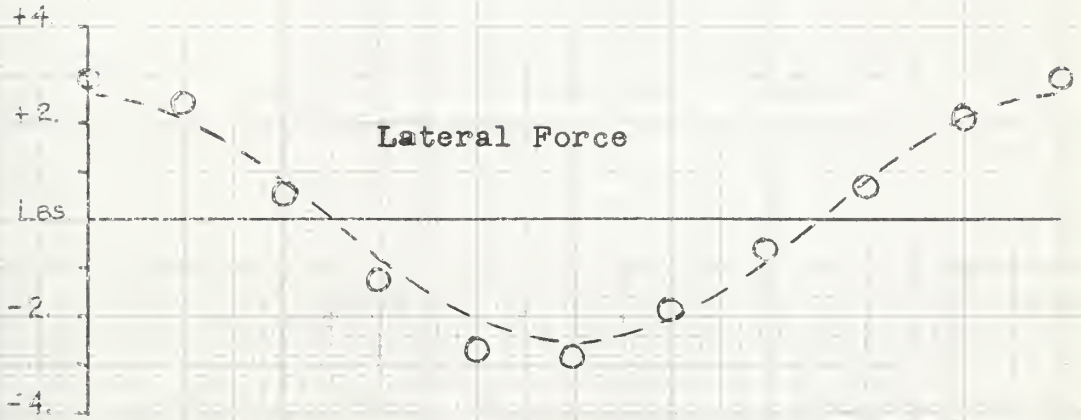


Submarine shape, 5 ft. wavelength, angle of attack 90°, axis 10" below still water

Run number 14

RJE

Run number 14 (continued)



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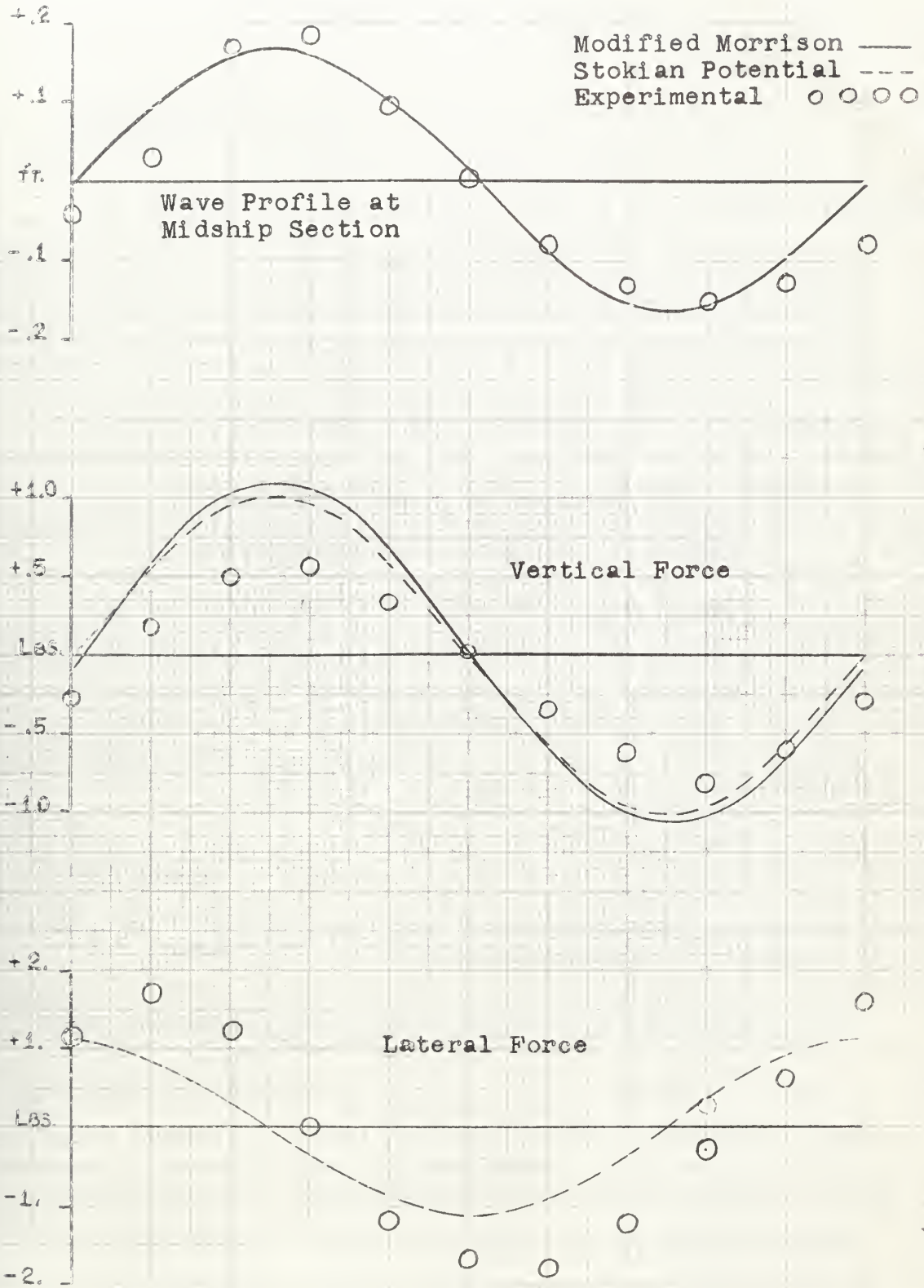

2 7/8" cylinder, 5 ft. wavelength, angle of attack 90°, axis 10" below still water

Run number 15

RJE

V. SUMMARY AND CONCLUSIONS

A. Summary

Laboratory measurements of forces and moments exerted by water waves on a stationary, submerged, horizontal submarine shape and two dimensional circular cylinders have been made at various relative depths and angles of incidence.

These results have been compared with two theories:

1. Potential theory of Cummins (6)
2. Superposition of inertial and drag forces after Morrison (3), modified by an incremental formulation which provides for variation in the force coefficients with axial body position.

B. Conclusions

The comparison of theory and experiment shows:

1. Both theories agree very closely for predominance of inertia forces.
2. Where inertial forces predominate the predicted phase relationship is excellent and the error between theory and experiment at maximum force has an average value of approximately 19% for the modified Morrison theory and 17% for the potential theory.

3. For the case of significant or predominant viscous forces the modified Morrison theory gives a very good prediction of phase relationship and yields an error in maximum force having an average value of approximately 22% for the modified Morrison theory and 32% for the potential flow theory.

4. Instrumentation difficulties are estimated to contribute heavily to the apparent error particularly with respect to measured wave height and vertical moment.

5. The true error between theory and experiment is believed due predominately to the use of inexact drag and inertia coefficients.

6. Additional experiments are needed to define the time variation of both the inertia and drag coefficients in cases of oscillatory flow.

VI. RECOMMENDATIONS FOR FURTHER STUDY

A. Additional experiments should be run covering a wider range of parameters in order to further verify the modified Morrison theory.

B. Any additional experiments should be run with an improved instrument for measuring overturning moments. It is recommended that a torsion type gage similar to the design presented in this report be employed for the direct measurement of overturning moment.

C. Further study of C_D and C_m should be made with the objective of obtaining accurate instantaneous values of these coefficients in oscillatory flow.

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VIII.

A P P E N D I C E S

APPENDIX A. TABLES OF RESULTS

The tables included here contain the experimental and theoretical results obtained for each test run. The tables are arranged in order of run number starting with run number 1 and continuing through run number 15. There are four* pages for each run number. Thus, the first four pages pertain to run number 1 and present the results in the following order: first page, experimental; second page, modified Morrison theory; third page, Stokian potential theory; and fourth page, sine wave potential theory. The next four pages pertain to run number 2 and so on through the Appendix. Each table is labeled at the top indicating the method and run number for quick identification.

The tables of experimental results are self-explanatory. However, as an aid to interpretation of the theoretical tables the following example is given.

Consider the Modified Morrison Solution - Stokian Wave, Run Number 1. The wave length (WL) equals 10.0 feet. The body length (BL) equals 2.5 feet. The wave height (H) equals .630 feet. The angle of attack (ALPHA) equals 0.00001 radians $\approx 0°$. The coefficients of the sixth degree polynomial describing the body are as follows: constant term = .3333 feet; first power coefficient = 0.0; etc. (in this example the model was the 4" cylinder).

* For exceptions, see note on next page.

At phase angle 0°, the vertical force = -.65337439 lbs.; the vertical moment = .75109979; the ratio of vertical drag force to total vertical force (Ratio) = .54955693; ETA = -3.5781317E-02; and celerity = 6.8191871. At phase angle 72°, the vertical force = 2.3215859; RATIO = -4.6022410E-02; and ETA = .32853091; and celerity = 6.8191871.

Although there are some slight variations, the format of the other tables is essentially the same and should be interpreted similarly. These tables have been printed directly from the computer output and therefore some of the numbers are in floating point form. For example: 9.66E-06 should be interpreted as 9.66×10^{-6}.

Note: Run numbers 2, 6, 12, and 14 have seven pages each since the theoretical calculations are included for both the stern first and bow first orientations.

EXPERIMENTAL RESULTS

Run Number 1

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
4" cyl.	0°	10'	7.56"	10.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	-0.135	-1.500	0.446	1.130
36	0.150	0.500	0.330	1.130
72	0.312	2.300	0.087	0.602
108	0.322	2.675	0.136	-0.204
144	0.200	1.475	0.320	-1.020
180	-0.035	-0.440	0.048	-1.025
216	-0.212	-1.920	-0.019	-1.130
252	-0.272	-3.010	-0.015	-0.527
288	-0.278	-2.900	0.175	0.145
324	-0.235	-2.590	0.300	0.806
360	-0.105	-1.275	0.446	1.130

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 1

WL	BL	H	DEPTH	z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.630	2.0	-.8333	1.93	9.66E-06	.00001
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA -- CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-.65337449	3.8604371E-05	.75109979	6.5102020E-07
.54955693			
-3.5781317E-02	6.8191871		
.96905360	3.2009790E-05	.96520920	-7.5494290E-06
-.30307991			
.17409580	6.8191871		
2.3215859	1.2411131E-05	.48378401	-1.3067379E-05
-4.6022410E-02			
.32853091	6.8191871		
2.5352703	-1.2408610E-05	-.27777548	-1.3067589E-05
4.2144374E-02			
.32852988	6.8191871		
1.5564387	-3.2007922E-05	-.75321598	-7.5499118E-06
.18870160			
.17409343	6.8191871		
6.4741400E-02	-3.8603406E-05	-.82226051	6.5054670E-07
5.5461631			
-3.5783628E-02	6.8191871		
-1.2178897	-3.0453386E-05	-.52499325	7.9517624E-06
-.18619499			
-.19621104	6.8191871		
-1.8881707	-1.1448547E-05	-.16734061	1.2014537E-05
-3.3927206E-02			
-.27063568	6.8191871		
-2.0162881	1.1448860E-05	4.3490840E-02	1.2014613E-05
3.1773741E-02			
-.27063446	6.8191871		
-1.6713970	3.0453931E-05	.30233913	7.9519432E-06
.13567769			
-.19620692	6.8191871		
-.65334169	3.8604397E-05	.75111003	6.5082500E-07
.54958608			
-3.5776692E-02	6.8191871		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER 1

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.630	2.0	-.8333	1.93	9.66E-06	0.0
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-.27774295	.74240273	.00000000	.00000000
1.7389183			
-3.5781317E-02			
1.1916802	.81085038	.00000000	.00000000
1.4418401			
.17409580			
2.2917469	.35934393	.00000000	.00000000
.55899823			
.32853091			
2.2917395	-.35935177	.00000000	.00000000
-.55901084			
.32852988			
1.1916634	-.81085250	.00000000	.00000000
-1.4418474			
.17409343			
-.27775891	-.74239937	.00000000	.00000000
-1.7389177			
-3.5783628E-02			
-1.3633423	-.39037691	.00000000	.00000000
-1.3717781			
-.19621104			
-1.8423499	-9.9479440E-02	.00000000	.00000000
-.51569103			
-.27063568			
-1.8423424	9.9487100E-02	.00000000	.00000000
.51572614			
-.27063446			
-1.3633152	.39038952	.00000000	.00000000
1.3718012			
-.19620692			
-.27771099	.74240911	.00000000	.00000000
1.7389193			
-3.5776692E-02			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 1

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.630	2.0	-.8333	1.93	9.66E-06	0.0
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
.00000000	.00000000	.97663094	.00000000
.00000000			
1.6805604	.00000000	.79011016	.00000000
.18515273			
2.7192010	.00000000	.30179281	.00000000
.29958309			
2.7191944	.00000000	-.30179960	.00000000
.29958238			
1.6805434	.00000000	-.79011431	.00000000
.18515086			
-2.0960000E-05	.00000000	-.97663080	.00000000
-2.3114700E-06			
-1.6805774	.00000000	-.79010587	.00000000
-.18515460			
-2.7192075	.00000000	-.30178599	.00000000
-.29958380			
-2.7191880	.00000000	.30180641	.00000000
-.29958166			
-1.6805264	.00000000	.79011851	.00000000
-.18514898			
4.1970000E-05	.00000000	.97663081	.00000000
4.6251450E-06			
7.1558506			

EXPERIMENTAL RESULTS

Run Number 2

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
Sub	0°	10"	7.56"	10.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	-0.095	-0.570	0.317	1.470
36	0.130	1.400	0.154	1.290
72	0.308	2.750	0.154	0.462
108	0.322	2.840	0.212	-0.337
144	0.208	1.320	0.202	-1.345
180	-0.060	-1.090	-0.241	-1.654
216	-0.218	-2.630	-0.154	-1.240
252	-0.262	-3.490	-0.212	-0.442
288	-0.279	-3.050	-0.048	0.500
324	-0.222	-2.150	0.163	1.085
360	-0.055	-0.525	0.346	1.460

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 2

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.2917	.630	2.0	-.8333	1.93	9.66E-06	.00001
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force RATIO ETA - CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-1.0656721 .36256475	4.8203990E-05	.45588950	-4.9670603E-06
-3.5781317E-02 .97319690	6.8191871 4.2401360E-05	.36258410	-8.4332442E-06
-.34366181 .17409580	6.8191871		
2.9001994 -3.6396373E-02	1.9264074E-05	-8.2453650E-02	-8.6852926E-06
.32853091 3.3017310 2.1804841E-02	6.8191871 -1.2130750E-05	-.51195430	-5.2421597E-06
.32852988 2.1029941 .14030865	6.8191871 -3.8308826E-05	-.59806700	4.4381820E-07
.17409343 .16700561 2.3960077	6.8191871 -4.8594197E-05	-.41479092	5.7312159E-06
-3.5783628E-02 -1.4852996 -.16403790	6.8191871 -4.0122369E-05	-7.6636613E-02	8.4474016E-06
-.19621104 -2.2956511 -2.5290965E-02	6.8191871 -1.7463842E-05	.19791034	7.9298694E-06
-.27063568 -2.4445476 1.5987789E-02	6.8191871 1.0966052E-05	.30323009	4.7610783E-06
-.27063446 -2.1540432 9.4473629E-02	6.8191871 3.5790511E-05	.36423571	1.4199920E-08
-.19620692 -1.0656342 .36257994	6.8191871 4.8204080E-05	.45589055	-4.9671660E-06
-3.5776692E-02	6.8191871		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 2

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.2917	.630	2.0	-.8333	1.93	9.66E-06	0.0
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-.63008841	.43358893	.00000000	.00000000
2.1713363			
-3.5781317E-02			
1.2429275	.27736616	.00000000	.00000000
1.9099326			
.17409580			
2.8367305	-.12048792	.00000000	.00000000
.86769309			
.32853091			
3.0394626	-.51652822	.00000000	.00000000
-.54649164			
.32852988			
1.6956097	-.60690734	.00000000	.00000000
-1.7256698			
.17409343			
-.22669670	-.35428654	.00000000	.00000000
-2.1889513			
-3.5783628E-02			
-1.6340276	-5.9909950E-03	.00000000	.00000000
-1.8073175			
-.19621104			
-2.2216734	.20890660	.00000000	.00000000
-.78665051			
-.27063568			
-2.2682472	.29980174	.00000000	.00000000
.49397807			
-.27063446			
-1.8340290	.38454759	.00000000	.00000000
1.6121879			
-.19620692			
-.63005037	.43358829	.00000000	.00000000
2.1713403			
-3.5776692E-02			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 2

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.2917	.630	2.0	-.8333	1.93	9.66E-06	0.0
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
-.26534472	.00000000	.51822792	.00000000
.00000000			
1.8923101	.00000000	.18638054	.00000000
.18515273			
3.3271637	.00000000	-.21665824	.00000000
.29958309			
3.4911479	.00000000	-.53694054	.00000000
.29958238			
2.3216263	.00000000	-.65212885	.00000000
.18515086			
.26531839	.00000000	-.51822497	.00000000
-2.3114700E-06			
-1.8923326	.00000000	-.18637592	.00000000
-.18515460			
-3.3271738	.00000000	.21666276	.00000000
-.29958380			
-3.4911417	.00000000	.53694323	.00000000
-.29958166			
-2.3216062	.00000000	.65212873	.00000000
-.18514898			
-.26529207	.00000000	.51822206	.00000000
4.6251450E-06			
7.1558506			

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER 2

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.	2.2917	.630	2.0	-.8333	1.93	9.66E-06	.00001
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force RATIO ETA	Lateral Force CELERITY	Vertical Moment	Lateral Moment
-.63352014	4.8618656E-05	.31010247	5.7335997E-06
.63181168			
-3.5781317E-02	6.8191871		
1.5136521	3.8329450E-05	.67792726	4.4308343E-07
-.19499856			
.17409580	6.8191871		
3.1592984	1.2139486E-05	.59913571	-5.2458702E-06
-2.2796876E-02			
.32853091	6.8191871		
3.1128327	-1.9270587E-05	.21149196	-8.6904294E-06
3.3921681E-02			
.32852988	6.8191871		
1.6428603	-4.2419961E-05	-.18511827	-8.4375908E-06
.20363856			
.17409343	6.8191871		
-.29311979	-4.8226590E-05	-.44218832	-4.9689347E-06
-1.3185452			
-3.5783628E-02	6.8191871		
-1.7479567	-3.5807553E-05	-.45458902	1.5391470E-08
-.11645793			
-.19621104	6.8191871		
-2.3675716	-1.0971372E-05	-.35214078	4.7646544E-06
-1.6513415E-02			
-.27063568	6.8191871		
-2.4129613	1.7472348E-05	-.27155478	7.9346316E-06
2.4071234E-02			
-.27063446	6.8191871		
-1.9735026	4.0142184E-05	-9.3127519E-02	8.4517525E-06
.12349923			
-.19620692	6.8191871		
-.63347626	4.8618642E-05	.31011323	5.7335007E-06
.63185651			
-3.5776692E-02	6.8191871		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER 2

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.	2.2917	.630	2.0	-.8333	1.93	9.66E-06	0.0
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-.22680704	.35453098	.00000000	.00000000
2.1900142			
-3.5781317E-02			
1.6964243	.60724320	.00000000	.00000000
1.7265114			
.17409580			
3.0409256	.51675324	.00000000	.00000000
.54676452			
.32853091			
2.8381064	.12046094	.00000000	.00000000
-.86810571			
.32852988			
1.2435468	-.27759480	.00000000	.00000000
-1.9108547			
.17409343			
-.63037451	-.43385205	.00000000	.00000000
-2.1723918			
-3.5783628E-02			
-1.8349311	-.38473321	.00000000	.00000000
-1.6129563			
-.19621104			
-2.2693604	-.29991471	.00000000	.00000000
-.49419633			
-.27063568			
-2.2227551	-.20895651	.00000000	.00000000
.78705478			
-.27063446			
-1.6347997	6.0669590E-03	.00000000	.00000000
1.8082097			
-.19620692			
-.22676511	.35453917	.00000000	.00000000
2.1900135			
-3.5776692E-02			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER 2

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.	2.2917	.630	2.0	-.8333	1.93	9.66E-06	0.0
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
.26543406	.00000000	.51855884	.00000000
.00000000			
2.3227435	.00000000	.65247326	.00000000
.18515273			
3.4928399	.00000000	.53716397	.00000000
.29958309			
3.3287841	.00000000	.21667537	.00000000
.29958238			
1.8932403	.00000000	-.18657623	.00000000
.18515086			
-.26546036	.00000000	-.51856171	.00000000
-2.3114700E-06			
-2.3227638	.00000000	-.65247338	.00000000
-.18515460			
-3.4928463	.00000000	-.53716124	.00000000
-.29958380			
-3.3287743	.00000000	-.21667086	.00000000
-.29958166			
-1.8932177	.00000000	.18658081	.00000000
-.18514898			
.26548670	.00000000	.51856463	.00000000
4.6251450E-06			
7.1558506			

EXPERIMENTAL RESULTS

Run Number 3

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
2-7/8" cyl.	0°	10"	7.56"	10.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft. lbs.	Axial Force lbs.
0	0.0	-0.290	1.468	.762
36	.190	0.280	1.280	.618
72	.330	0.720	0.656	.276
108	.320	0.750	-0.143	.352
144	.210	0.388	-0.823	-.695
180	-.081	-0.170	-1.420	-.809
216	-.220	-0.670	-0.800	-.685
252	-.290	-0.990	-0.234	-.209
288	-.272	-0.980	0.540	.209
324	-.177	-0.720	1.132	.666
360	.010	-0.260	1.448	.762

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 3

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.630	2.0	-.8333	1.93	9.66E-06	.00001
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
RATIO			
ETA - CELERITY			
-.39203607	1.9946700E-05	.37339746	3.3649450E-07
.62068862			
-3.5781317E-02	6.8191871		
.43882926	1.6539560E-05	.50582432	-3.9006067E-06
-.45392855			
.17409580	6.8191871		
1.1545002	6.4132741E-06	.26227931	-6.7517396E-06
-6.2800054E-02			
.32853091	6.8191871		
1.2995029	-6.4108394E-06	-.12251047	-6.7519385E-06
5.5793746E-02			
.32852988	6.8191871		
.83721586	-1.6537751E-05	-.36243717	-3.9010721E-06
.23792960			
.17409343	6.8191871		
9.4619590E-02	-1.9945789E-05	-.42156760	3.3601190E-07
2.5716860			
-3.5783628E-02	6.8191871		
-.57602820	-1.5734955E-05	-.28428180	4.1085050E-06
-.26718520			
-.19621104	6.8191871		
-.94286363	-5.9154609E-06	-9.5429470E-02	6.2077567E-06
-4.6169795E-02			
-.27063568	6.8191871		
-1.0299263	5.9154017E-06	1.1096180E-02	6.2078659E-06
4.2269975E-02			
-.27063446	6.8191871		
-.88383029	1.5735248E-05	.13374111	4.1087812E-06
.17414023			
-.19620692	6.8191871		
-.39201959	1.9946711E-05	.37340348	3.3639420E-07
.62071625			
-3.5776692E-02	6.8191871		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 3

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.630	2.0	-.8333	1.93	9.66E-06	0.0
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force AXIAL FORCE ETA	Vertical Moment	Lateral Force	Lateral Moment
-.14350714 .89848263 -3.5781317E-02	.38359240	.00000000	.00000000
.61572987 .74498511 .17409580	.41895870	.00000000	.00000000
1.1841239 .28882908 .32853091	.18566960	.00000000	.00000000
1.1841201 -.28883558 .32852988	-.18567363	.00000000	.00000000
.61572125 -.74498891 .17409343	-.41895981	.00000000	.00000000
-.14351541 -.89848235 -3.5783628E-02	-.38359071	.00000000	.00000000
-.70442608 -.70878479 -.19621104	-.20170405	.00000000	.00000000
-.95192484 -.26645266 -.27063568	-5.1400060E-02	.00000000	.00000000
-.95192093 .26647081 -.27063446	5.1404050E-02	.00000000	.00000000
-.70441208 .70879671 -.19620692	.20171057	.00000000	.00000000
-.14349064 .89848320 -3.5776692E-02	.38359573	.00000000	.00000000

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER 3

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.630	2.0	-.8333	1.93	9.66E-06	0.0
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
.00000000	.00000000	.50461590	.00000000
.00000000			
.86832965	.00000000	.40824242	.00000000
.18515273			
1.4049854	.00000000	.15593347	.00000000
.29958309			
1.4049820	.00000000	-.15593699	.00000000
.29958238			
.86832093	.00000000	-.40824456	.00000000
.18515086			
-1.0840000E-05	.00000000	-.50461586	.00000000
-2.3114700E-06			
-.86833847	.00000000	-.40824020	.00000000
-.18515460			
-1.4049887	.00000000	-.15592996	.00000000
-.29958380			
-1.4049786	.00000000	.15594051	.00000000
-.29958166			
-.86831212	.00000000	.40824674	.00000000
-.18514898			
2.1690000E-05	.00000000	.50461584	.00000000
4.6251450E-06			
7.1558506			

EXPERIMENTAL RESULTS

Run Number 4

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
1" cyl.	0°	10"	9.6"	10'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	0.045	-0.080	0.143	0.085
36	0.255	-0.009	0.139	0.065
72	0.428	0.060	0.100	0.018
108	0.450	0.090	-0.024	-0.038
144	0.150	0.082	-0.069	-0.097
180	-0.190	0.055	-0.144	-0.099
216	-0.318	0.010	-0.126	-0.073
252	-0.340	-0.057	-0.049	-0.022
288	-0.290	-0.110	0.022	0.035
324	-0.203	-0.128	0.091	0.073
360	0.002	-0.094	0.137	0.087

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER 4

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.800	2.0	-.8333	1.93	9.66E-06	.00001
.0833	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA - CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-.13703102	3.1822275E-06	8.0940480E-03	6.8420640E-08
.91883421			
-5.7697260E-02	6.9512435		
-6.9746400E-02	2.6565185E-06	6.3173469E-02	-6.1749196E-07
1.5283408			
.21728531	6.9512435		
3.4180245E-02	1.0349533E-06	5.1245906E-02	-1.0885991E-06
-1.1703612			
.42710123	6.9512435		
.11418739	-1.0320273E-06	2.6370342E-02	-1.0888208E-06
.35033788			
.42709983	6.9512435		
.14344634	-2.6543086E-06	8.4539180E-03	-6.1802774E-07
.74311432			
.21728213	6.9512435		
.11478574	-3.1811167E-06	-3.8730379E-02	6.7796940E-08
1.0969000			
-5.7700195E-02	6.9512435		
3.3713288E-02	-2.4926186E-06	-4.8941810E-02	6.5960184E-07
2.2969364			
-.25294542	6.9512435		
-3.4872173E-02	-9.3317870E-07	-2.2958604E-02	9.7841740E-07
-.61123604			
-.33374512	6.9512435		
-7.7503858E-02	9.3261365E-07	-1.8896064E-02	9.7861106E-07
.27504095			
-.33374389	6.9512435		
-.12116277	2.4926799E-06	-2.7898188E-02	6.6010112E-07
.63913638			
-.25294070	6.9512435		
-.13703047	3.1822300E-06	8.0954960E-03	6.8404630E-08
.91884118			
-5.7691390E-02	6.9512435		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 4

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.800	2.0	-.8333	1.93	9.66E-06	0.0
.0833	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-2.9068397E-02	6.1188264E-02	.00000000	.00000000
.14332031			
-5.7697260E-02			
9.6308583E-02	7.1505311E-02	.00000000	.00000000
.11961437			
.21728531			
.19388138	3.2506532E-02	.00000000	.00000000
4.6553658E-02			
.42710123			
.19388071	-3.2507236E-02	.00000000	.00000000
-4.6554701E-02			
.42709983			
9.6307110E-02	-7.1505466E-02	.00000000	.00000000
-.11961497			
.21728213			
-2.9069716E-02	-6.1187916E-02	.00000000	.00000000
-.14332026			
-5.7700195E-02			
-.11427434	-2.7498898E-02	.00000000	.00000000
-.11228190			
-.25294542			
-.14684767	-5.3094010E-03	.00000000	.00000000
-4.2021420E-02			
-.33374512			
-.14684721	5.3098540E-03	.00000000	.00000000
4.2024287E-02			
-.33374389			
-.11427236	2.7500006E-02	.00000000	.00000000
.11228381			
-.25294070			
-2.9065767E-02	6.1188935E-02	.00000000	.00000000
.14332043			
-5.7691390E-02			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 4

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
10.0	2.5	.800	2.0	-.8333	1.93	9.66E-06	0.0
.0833	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment

ETA

CELERITY (Last Line Only)

.00000000	.00000000	7.7463886E-02	.00000000
.00000000			
.13329779	.00000000	6.2669533E-02	.00000000
.23511458			
.21568014	.00000000	2.3937438E-02	.00000000
.38042297			
.21567961	.00000000	-2.3937983E-02	.00000000
.38042206			
.13329644	.00000000	-6.2669863E-02	.00000000
.23511220			
-1.6640000E-06	.00000000	-7.7463879E-02	.00000000
-2.9352000E-06			
-.13329914	.00000000	-6.2669194E-02	.00000000
-.23511695			
-.21568064	.00000000	-2.3936898E-02	.00000000
-.38042388			
-.21567909	.00000000	2.3938523E-02	.00000000
-.38042115			
-.13329509	.00000000	6.2670197E-02	.00000000
-.23510982			
3.3290000E-06	.00000000	7.7463881E-02	.00000000
5.8732000E-06			
7.1558506			

EXPERIMENTAL RESULTS

Run Number 5

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
4" cyl.	0°	10"	4.0"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	-.070	-0.175	1.830	0.679
36	.040	0.350	1.360	0.547
72	.158	0.970	0.630	0.264
108	.168	1.120	-0.284	-0.0943
144	.060	0.640	-1.138	-0.472
180	-.035	-0.065	-1.495	-0.604
216	-.118	-0.735	-1.335	-0.475
252	-.155	-1.100	-0.619	-0.208
288	-.155	-1.100	0.402	0.160
324	-.148	-0.740	1.138	0.472
360	-.070	-0.130	1.415	0.688

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 5

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.00001
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA - CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-5.5712911E-02 1.0001315	1.5377233E-05	1.0730748	-8.9955500E-08
-2.9928239E-03 .74230871	5.1401008 1.2440683E-05	.90095644	-7.2213753E-06
-6.7774962E-02 9.6941615E-02 1.2598317	5.1401008 4.7521082E-06	.38122449	-1.1566617E-05
-1.7971591E-02 .16077232 1.3051116	5.1401008 -4.7516532E-06	-.28825441	-1.1566681E-05
1.7348490E-02 .16077191 .84292078	5.1401008 -1.2440323E-05	-.84558410	-7.2215477E-06
5.9685419E-02 9.6940590E-02 5.5717684E-02 1.0000466	5.1401008 -1.5377035E-05	-1.0734108	-9.0169010E-08
-2.9940456E-03 -.74229238	5.1401008 -1.2440176E-05	-.89111583	7.1657602E-06
-6.7803181E-02 -9.8792210E-02 -1.2598694	5.1401008 -4.7516563E-06	-.37470876	1.1712441E-05
-1.7952980E-02 -.15593016 -1.3050985	5.1401008 4.7518108E-06	.28241676	1.1712450E-05
1.7331879E-02 -.15592911 -.84292842	5.1401008 1.2440353E-05	.83543673	7.1657824E-06
5.9708893E-02 -9.8789370E-02 -5.5693138E-02 1.0004865	5.1401008 1.5377235E-05	1.0730756	-9.0135050E-08
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 5

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	0.0
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
MFA			
6.7400000E-06	.99103079	.00000000	.00000000
.69266406			
-2.9928241E-03			
.73190297	.80637427	.00000000	.00000000
.56038181			
9.6941615E-02			
1.1842336	.30909443	.00000000	.00000000
.21404637			
.16077232			
1.1842309	-.30910137	.00000000	.00000000
-.21405121			
.16077192			
.73189555	-.80637854	.00000000	.00000000
-.56038478			
9.6940590E-02			
-2.3700000E-06	-.99103072	.00000000	.00000000
-.69266407			
-2.9940458E-03			
-.73190621	-.79714128	.00000000	.00000000
-.56036805			
-9.8792210E-02			
-1.1842475	-.30338394	.00000000	.00000000
-.21403488			
-.15593016			
-1.1842390	.30340452	.00000000	.00000000
.21404938			
-.15592911			
-.73188400	.79715417	.00000000	.00000000
.56037700			
-9.8789370E-02			
2.5020000E-05	.99103096	.00000000	.00000000
.69266409			
-2.9903794E-03			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 5

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	0.0
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only			
.00000000	.00000000	1.0076330	.00000000
.00000000			
.74416202	.00000000	.81519140	.00000000
9.7866442E-02			
1.2040781	.00000000	.31137293	.00000000
.15835106			
1.2040753	.00000000	-.31137994	.00000000
.15835068			
.74415453	.00000000	-.81519575	.00000000
9.7865453E-02			
-9.2800000E-06	.00000000	-1.0076330	.00000000
-1.2217770E-06			
-.74416955	.00000000	-.81518706	.00000000
-9.7867431E-02			
-1.2040810	.00000000	-.31136588	.00000000
-.15835144			
-1.2040724	.00000000	.31138699	.00000000
-.15835030			
-.74414699	.00000000	.81520009	.00000000
-9.7864463E-02			
1.8600000E-05	.00000000	1.0076330	.00000000
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 6

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
Sub	0°	10"	4.0"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	-.022	-0.040	1.392	1.06
36	.040	1.020	1.150	0.802
72	.161	1.640	0.570	0.198
108	.181	1.500	-0.014	-0.556
144	.085	0.750	-0.717	-1.038
180	-.008	0.345	-1.185	-1.114
216	-.090	-1.260	-0.915	-0.778
252	-.150	-1.690	-0.445	-0.189
288	-.148	-1.600	0.291	0.387
324	-.135	-1.080	1.000	0.935
360	-.060	-0.130	1.480	1.114

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 6

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	.00001
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force RATIO ETA - CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-.40725375 .18464792 -2.9928239E-03	2.3764159E-05 5.1401008	.63397782	-2.2867374E-06
.89246210 -7.5641479E-02	2.1253247E-05	.42649336	-6.0284617E-06
9.6941615E-02 1.8580283 -1.7233900E-02	5.1401008 1.0720330E-05	4.0420009E-02	-7.4384296E-06
.16077232 2.1113068 8.0573737E-03	5.1401008 -3.8011795E-06	-.36022288	-6.0808811E-06
.16077191 1.5458420 3.5188132E-02	5.1401008 -1.6901116E-05	-.61203425	-2.4752897E-06
9.6940590E-02 .39371292 .19129793	5.1401008 -2.3670300E-05	-.63443794	2.1033853E-06
-2.9940456E-03 -.90219443 -7.4545858E-02	5.1401008 -2.1445010E-05	-.41643671	5.9703338E-06
-9.8792210E-02 -1.8505985 -1.7110701E-02	5.1401008 -1.0932372E-05	-3.3762478E-02	7.5858821E-06
-.15593016 -2.0968439 8.0280167E-03	5.1401008 3.8623144E-06	.35427795	6.2301523E-06
-.15592911 -1.5444870 3.5018342E-02	5.1401008 1.7151367E-05	.60174173	2.4200456E-06
-9.8789370E-02 -.40722337 .18466219	5.1401008 2.3764212E-05	.63397568	-2.2868433E-06
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 6

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	0.0
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-.30402612	.58181536	.00000000	.00000000
1.0704540			
-2.9928241E-03			
.88362727	.37093279	.00000000	.00000000
.95734464			
9.6941615E-02			
1.7380959	1.3682874E-02	.00000000	.00000000
.48288528			
.16077232			
1.9250902	-.35172981	.00000000	.00000000
-.17123713			
.16077192			
1.3702185	-.57992482	.00000000	.00000000
-.76132099			
9.6940590E-02			
.29150092	-.58189824	.00000000	.00000000
-1.0662339			
-2.9940458E-03			
-.89230506	-.36156328	.00000000	.00000000
-.96599082			
-9.8792210E-02			
-1.7309466	-7.8057940E-03	.00000000	.00000000
-.49244525			
-.15593016			
-1.9120005	.34599696	.00000000	.00000000
.17398204			
-.15592911			
-1.3692757	.57050831	.00000000	.00000000
.77258517			
-9.8789370E-02			
-.30399799	.58181291	.00000000	.00000000
1.0704564			
-2.9903794E-03			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 6

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	0.0
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
-.30275899	.00000000	.59160504	.00000000
.00000000			
.90283537	.00000000	.37238541	.00000000
9.7866442E-02			
1.7635755	.00000000	1.0926558E-02	.00000000
.15835106			
1.9506870	.00000000	-.35470589	.00000000
.15835068			
1.3926989	.00000000	-.58485213	.00000000
9.7865453E-02			
.30274462	.00000000	-.59160373	.00000000
-1.2217770E-06			
-.90284827	.00000000	-.37238179	.00000000
-9.7867431E-02			
-1.7635821	.00000000	-1.0922017E-02	.00000000
-.15835144			
-1.9506846	.00000000	.35470961	.00000000
-.15835030			
-1.3926885	.00000000	.58485362	.00000000
-9.7864463E-02			
-.30273029	.00000000	.59160239	.00000000
2.4447195E-06			
5.0599504			

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER 6

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	.00001
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force RATIO ETA	Lateral Force CELERITY	Vertical Moment	Lateral Moment
.24317916	2.3681461E-05	.62698556	2.1041582E-06
-.30979921			
-2.9928239E-03	5.1401008		
1.4377068	1.6909574E-05	.64723345	-2.4772545E-06
-3.7846560E-02			
9.6941615E-02	5.1401008		
2.0782457	3.8038297E-06	.40566327	-6.0848391E-06
-8.1886751E-03			
.16077232	5.1401008		
1.9229715	-1.0724437E-05	1.4144851E-02	-7.4428914E-06
1.6656603E-02			
.16077191	5.1401008		
1.0279684	-2.1262513E-05	-.37553658	-6.0317703E-06
6.5688417E-02			
9.6940590E-02	5.1401008		
-.25696082	-2.3774998E-05	-.62728462	-2.2876035E-06
-.29272429			
-2.9940456E-03	5.1401008		
-1.4369942	-1.7159291E-05	-.63707673	2.4221004E-06
-3.7649103E-02			
-9.8792210E-02	5.1401008		
-2.0641397	-3.8642422E-06	-.39913365	6.2342411E-06
-8.1578538E-03			
-.15593016	5.1401008		
-1.9148136	1.0937246E-05	-2.0227219E-02	7.5904264E-06
1.6542120E-02			
-.15592911	5.1401008		
-1.0371724	2.1454866E-05	.36524532	5.9735358E-06
6.4862716E-02			
-9.8789370E-02	5.1401008		
.24321012	2.3681405E-05	.62698920	2.1040531E-06
-.30975883			
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER 6

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	0.0

.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
.29160968	.58226004	.00000000	.00000000
1.0667290			
-2.9928241E-03			
1.3708319	.58024713	.00000000	.00000000
.76168365			
9.6941615E-02			
1.9259741	.35188697	.00000000	.00000000
.17133102			
.16077232			
1.7389105	-1.3752260E-02	.00000000	.00000000
-.48309554			
.16077192			
.88405976	-.37120049	.00000000	.00000000
-.95778034			
9.6940590E-02			
-.30413929	-.58217668	.00000000	.00000000
-1.0709503			
-2.9940458E-03			
-1.3699091	-.57082253	.00000000	.00000000
-.77294061			
-9.8792210E-02			
-1.9128844	-.34614419	.00000000	.00000000
-.17406119			
-.15593016			
-1.7317457	7.8805147E-03	.00000000	.00000000
.49267300			
-.15592911			
-.89271518	.36183327	.00000000	.00000000
.96643675			
-9.8789370E-02			
.29163802	.58226279	.00000000	.00000000
1.0667263			
-2.9903794E-03			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER 6

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	0.0
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
.30285746	.00000000	.59197134	.00000000
.00000000			
1.3933225	.00000000	.58517722	.00000000
9.7866442E-02			
1.9515833	.00000000	.35486429	.00000000
.15835106			
1.7644023	.00000000	-1.0995362E-02	.00000000
.15835068			
.90327658	.00000000	-.37265512	.00000000
9.7865453E-02			
-.30287180	.00000000	-.59197267	.00000000
-1.2217770E-06			
-1.3933328	.00000000	-.58517575	.00000000
-9.7867431E-02			
-1.9515858	.00000000	-.35486054	.00000000
-.15835144			
-1.7643958	.00000000	1.0999907E-02	.00000000
-.15835030			
-.90326363	.00000000	.37265877	.00000000
-9.7864463E-02			
.30288615	.00000000	.59197399	.00000000
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 7

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
2-7/8" cyl.	0°	6"	4.0"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	-.045	-0.180	0.970	0.364
36	.050	0.365	1.058	0.236
72	.160	0.860	0.525	0.145
108	.178	0.940	-0.125	-0.0454
144	.082	0.620	-0.694	-0.291
180	-.011	-0.0	-0.940	-0.364
216	-.099	-0.670	-0.832	-0.309
252	-.155	-1.110	-0.402	-0.147
288	-.157	-1.170	0.206	0.0836
324	-.138	-0.850	0.820	0.262
360	-.095	-0.200	1.040	0.346

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 7

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.50	1.93	9.66E-06	.00001
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA- CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-.10045818 1.0000865 -2.9928239E-03	1.1731923E-05 5.1401008	.85684437	-9.7913000E-09
.54235583 -.16737838 9.6941615E-02	9.4915410E-06 5.1401008	.74933124	-5.4911659E-06
.98350970 -4.1596958E-02 .16077232	3.6257727E-06 5.1401008	.35258253	-8.8720026E-06
1.0653301 3.8403023E-02 .16077191	-3.6248424E-06 5.1401008	-.18454019	-8.8721240E-06
.72390724 .12540162 9.6940590E-02	-9.4907862E-06 5.1401008	-.64958780	-5.4914871E-06
.10046756 .99999253 -2.9940456E-03	-1.1731451E-05 5.1401008	-.85774119	-1.0177070E-08
-.54230327 -.16749355 -9.8792210E-02	-9.4909970E-06 5.1401008	-.73796280	5.4850365E-06
-.98358730 -4.1531486E-02 -.15593016	-3.6252022E-06 5.1401008	-.34440498	8.8882383E-06
-1.0652823 3.8348995E-02 -.15592911	3.6253802E-06 5.1401008	.17814737	8.8883073E-06
-.72394949 .12546893 -9.8789370E-02	9.4912768E-06 5.1401008	.63738151	5.4852326E-06
-.10044247 1.0002437 -2.9903792E-03	1.1731925E-05 5.1401008	.85684566	-9.9280300E-09

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 7

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.50	1.93	9.66E-06	0.0
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force AXIAL FORCE ETA	Vertical Moment	Lateral Force	Lateral Moment
8.1300000E-06 .52845497 -2.9928241E-03	.80327558	.00000000	.00000000
.59324146 .42752920 9.6941615E-02	.65538552	.00000000	.00000000
.95987326 .16330047 .16077232	.25163669	.00000000	.00000000
.95987095 -.16330415 .16077192	-.25164238	.00000000	.00000000
.59323546 -.42753148 9.694090E-02	-.65538900	.00000000	.00000000
6.9000000E-07 -.52845497 -2.9940458E-03	-.80327549	.00000000	.00000000
-.59324246 -.42752572 -9.8792210E-02	-.64433682	.00000000	.00000000
-.95988866 -.16329602 -.15593016	-.24480489	.00000000	.00000000
-.95988179 .16330710 -.15592911	.24482150	.00000000	.00000000
-.59322446 .42753258 -9.8789370E-02	.64434728	.00000000	.00000000
2.2940000E-05 .52845498 -2.9903794E-03	.80327575	.00000000	.00000000

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 7

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.50	1.93	9.66E-06	0.0
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment

ETA

CELERITY (Last Line Only)

.00000000	.00000000	.79146542	.00000000
.00000000			
.58451686	.00000000	.64030830	.00000000
9.7866442E-02			
.94576717	.00000000	.24457406	.00000000
.15835106			
.94576490	.00000000	-.24457957	.00000000
.15835068			
.58451098	.00000000	-.64031174	.00000000
9.7865453E-02			
-7.2900000E-06	.00000000	-.79146543	.00000000
-1.2217770E-06			
-.58452278	.00000000	-.64030489	.00000000
-9.7867431E-02			
-.94576943	.00000000	-.24456853	.00000000
-.15835144			
-.94576266	.00000000	.24458509	.00000000
-.15835030			
-.58450507	.00000000	.64031514	.00000000
-9.7864463E-02			
1.4590000E-05	.00000000	.79146544	.00000000
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 8

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
2-7/8" cyl.	0°	10"	4.0"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	-.045	-0.115	0.703	0.373
36	.058	0.210	0.621	0.318
72	.170	0.430	0.362	0.147
108	.170	0.460	-0.0572	-0.0772
144	.720	0.310	-0.424	-0.262
180	0.000	-0.0	-0.500	-0.364
216	-.105	-0.34	-0.460	-0.241
252	-.155	-0.570	-0.225	-0.147
288	-.157	-0.630	0.1355	0.059
324	-.138	-0.490	0.542	0.262
360	-.068	-0.200	0.697	0.350

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 8

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.00001
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA-CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-3.8186266E-02 1.0000982	7.9453148E-06	.55121012	-4.6406470E-08
-2.9928239E-03 .37272200	5.1401008 6.4280829E-06	.46759073	-3.7311588E-06
-9.2413739E-02 9.6941615E-02 .64331278	5.1401008 2.4555025E-06	.20377983	-5.9763409E-06
-2.4080146E-02 .16077232 .67429364	5.1401008 -2.4550004E-06	-.14013014	-5.9764138E-06
2.2974254E-02 .16077191 .44160723	5.1401008 -6.4276956E-06	-.42960376	-3.7313612E-06
7.7998510E-02 9.6940590E-02 3.8188677E-02 1.0000345	5.1401008 -7.9451227E-06	-.55143539	-4.6662200E-08
-2.9940456E-03 -.37271072	5.1401008 -6.4277262E-06	-.46256107	3.7024194E-06
-9.2451711E-02 -9.8792210E-02 -.64333724	5.1401008 -2.4551783E-06	-.20037928	6.0516824E-06
-2.4053078E-02 -.15593016 -.67428197	5.1401008 2.4551718E-06	.13718487	6.0517352E-06
2.2950594E-02 -.15592911 -.44161413	5.1401008 6.4278051E-06	.42436576	3.7025515E-06
7.8027478E-02 -9.8789370E-02 -3.8176093E-02 1.0003646	5.1401008 7.9453158E-06	.55121060	-4.6499130E-08
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 8

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	0.0
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
3.5100000E-06	.51205618	.00000000	.00000000
.35789295			
-2.9928241E-03			
.37816729	.41664592	.00000000	.00000000
.28954396			
9.6941615E-02			
.61188234	.15970615	.00000000	.00000000
.11059572			
.16077232			
.61188085	-.15970975	.00000000	.00000000
-.11059823			
.16077192			
.37816347	-.41664813	.00000000	.00000000
-.28954552			
9.6940590E-02			
-1.2100000E-06	-.51205618	.00000000	.00000000
-.35789294			
-2.9940458E-03			
-.37816898	-.41187533	.00000000	.00000000
-.28953685			
-9.8792210E-02			
-.61188945	-.15675560	.00000000	.00000000
-.11058978			
-.15593016			
-.61188510	.15676623	.00000000	.00000000
.11059729			
-.15592911			
-.37815752	.41188198	.00000000	.00000000
.28954148			
-9.8789370E-02			
1.2930000E-05	.51205628	.00000000	.00000000
.35789297			
-2.9903794E-03			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 8

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	0.0
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
.00000000	.00000000	.52063444	.00000000
.00000000			
.38450145	.00000000	.42120165	.00000000
9.7866442E-02			
.62213577	.00000000	.16088344	.00000000
.15835106			
.62213428	.00000000	-.16088707	.00000000
.15835068			
.38449757	.00000000	-.42120390	.00000000
9.7865453E-02			
-4.7900000E-06	.00000000	-.52063443	.00000000
-1.2217770E-06			
-.38450534	.00000000	-.42119942	.00000000
-9.7867431E-02			
-.62213725	.00000000	-.16087980	.00000000
-.15835144			
-.62213279	.00000000	.16089070	.00000000
-.15835030			
-.38449369	.00000000	.42120614	.00000000
-9.7864463E-02			
9.6000000E-06	.00000000	.52063443	.00000000
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 9

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
2-7/8" cyl.	0°	14"	4.0"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	-.044	-0.12	0.497	0.246
36	.050	0.220	0.434	0.205
72	.171	0.450	0.211	0.0953
108	.170	0.475	-0.0764	-0.0454
144	.075	0.305	-0.348	-0.200
180	-.008	-0.0	-0.364	-0.291
216	-.091	-0.300	-0.322	-0.232
252	-.150	-0.500	-0.159	-0.109
288	-.150	-0.525	0.120	0.0227
324	-.125	-0.350	0.400	0.1545
360	-.044	-0.090	0.440	0.232

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 9

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.3333	2.0	-1.1667	1.93	9.66E-06	.00001
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA-CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-1.3629710E-02 1.0001198	5.5779876E-06	.33903409	-6.2296580E-08
-2.9982189E-03 .23819585	5.1403027 4.5128549E-06	.28218506	-2.6286462E-06
-5.1261350E-02 9.7028120E-02 .39976126	5.1403027 1.7238892E-06	.11674675	-4.1716517E-06
-1.3512446E-02 .16091934 .41056391	5.1403027 -1.7235811E-06	-9.4175370E-02	-4.1717003E-06
1.3157189E-02 .16091893 .26261383	5.1403027 -4.5126304E-06	-.26864597	-2.6287874E-06
4.6495156E-02 9.7027090E-02 1.3629824E-02 1.0001109	5.1403027 -5.5779025E-06	-.33908964	-6.2479620E-08
-2.9994417E-03 -.23819433	5.1403027 -4.5125741E-06	-.27999279	2.5900991E-06
-5.1274583E-02 -9.8882050E-02 -.39976762	5.1403027 -1.7236630E-06	-.11532011	4.2726280E-06
-1.3505355E-02 -.15606845 -.41056307	5.1403027 1.7236025E-06	9.2862308E-02	4.2726685E-06
1.3151108E-02 -.15606740 -.26261347	5.1403027 4.5125797E-06	.26640541	2.5901974E-06
4.6507267E-02 -9.8879205E-02 -1.3623464E-02 1.0005784	5.1403027 5.5779886E-06	.33903427	-6.2361610E-08
-2.9957720E-03	5.1403027		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 9

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.3333	2.0	-1.1667	1.93	9.66E-06	0.0
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
1.4920000E-06	.31224070	.00000000	.00000000
.25125886			
-2.9982191E-03			
.23059796	.25362835	.00000000	.00000000
.20327622			
9.7028120E-02			
.37311299	9.7117562E-02	.00000000	.00000000
7.7644880E-02			
.16091934			
.37311209	-9.7119740E-02	.00000000	.00000000
-7.7646626E-02			
.16091893			
.23059564	-.25362969	.00000000	.00000000
-.20327731			
9.7027090E-02			
-1.4010000E-06	-.31224067	.00000000	.00000000
-.25125886			
-2.9994419E-03			
-.23059939	-.25158583	.00000000	.00000000
-.20326765			
-9.8882050E-02			
-.37311633	-9.5853898E-02	.00000000	.00000000
-7.7638500E-02			
-.15606845			
-.37311365	9.5860414E-02	.00000000	.00000000
7.7643771E-02			
-.15606740			
-.23059241	.25158989	.00000000	.00000000
.20327091			
-9.8879205E-02			
7.2440000E-06	.31224073	.00000000	.00000000
.25125886			
-2.9957722E-03			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 9

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-1.1667	1.93	9.66E-06	0.0
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
1.0000000E-09	.00000000	.34243582	.00000000
.00000000			
.25289736	.00000000	.27703611	.00000000
9.7866442E-02			
.40919610	.00000000	.10581753	.00000000
.15835106			
.40919511	.00000000	-.10581992	.00000000
.15835068			
.25289481	.00000000	-.27703760	.00000000
9.7865453E-02			
-3.1520000E-06	.00000000	-.34243583	.00000000
-1.2217770E-06			
-.25289992	.00000000	-.27703464	.00000000
-9.7867431E-02			
-.40919707	.00000000	-.10581514	.00000000
-.15835144			
-.40919413	.00000000	.10582231	.00000000
-.15835030			
-.25289224	.00000000	.27703905	.00000000
-9.7864463E-02			
6.3210000E-06	.00000000	.34243581	.00000000
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 10

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
1" cyl.	0°	10"	4"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Vertical Moment ft.lbs.	Axial Force lbs.
0	-.045	-0.020	0.050	0.043
36	.048	0.025	0.073	0.031
72	.160	0.057	0.001	0.007
108	.168	0.060	-0.018	-0.015
144	.075	0.054	-0.055	-0.036
180	-.006	0.025	-0.070	-0.054
216	-.124	-0.010	-0.080	-0.043
252	-.160	-0.054	-0.040	-0.025
288	-.163	-0.055	+0.019	0.007
324	-.155	-0.040	+0.046	0.031
360	-.045	-0.009	0.050	0.044

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 10

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.00001
.0833	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
RATIO ETA-CELERITY			
-1.2810525E-02	9.6059051E-07	6.3910638E-02	-5.4566180E-09
1.0000341			
-2.9928239E-03	5.1401008		
3.5845034E-02	7.7729245E-07	5.8332292E-02	-4.5094491E-07
-.31751491			
9.6941615E-02	5.1401008		
7.1318274E-02	2.9712008E-07	3.0427627E-02	-7.2244056E-07
-7.1439488E-02			
.16077232	5.1401008		
8.1508070E-02	-2.9650780E-07	-9.4617670E-03	-7.2252108E-07
6.2509647E-02			
.16077191	5.1401008		
5.8607244E-02	-7.7683522E-07	-4.5731380E-02	-4.5119486E-07
.19419706			
9.6940590E-02	5.1401008		
1.2810821E-02	-9.6039534E-07	-6.3982845E-02	-5.7943800E-09
1.0000119			
-2.9940456E-03	5.1401008		
-3.5841463E-02	-7.7706175E-07	-5.7770185E-02	4.4745134E-07
-.31765204			
-9.8792210E-02	5.1401008		
-7.1325904E-02	-2.9690855E-07	-2.9991294E-02	7.3152894E-07
-7.1337350E-02			
-.15593016	5.1401008		
-8.1502066E-02	2.9670088E-07	9.1620840E-03	7.3164625E-07
6.2434105E-02			
-.15592911	5.1401008		
-5.8610359E-02	7.7703604E-07	4.5102641E-02	4.4773135E-07
.19425258			
-9.8789370E-02	5.1401008		
-1.2809363E-02	9.6059066E-07	6.3910796E-02	-5.4678250E-09
1.0001259			
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 10

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	0.0
.0833	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
4.2300000E-07	6.1902265E-02	.00000000	.00000000
4.3265530E-02			
-2.9928241E-03			
4.5716489E-02	5.0368153E-02	.00000000	.00000000
3.5002849E-02			
9.6941615E-02			
7.3970203E-02	1.9306810E-02	.00000000	.00000000
1.3369871E-02			
.16077232			
7.3970023E-02	-1.9307244E-02	.00000000	.00000000
-1.3370174E-02			
.16077192			
4.5716027E-02	-5.0368419E-02	.00000000	.00000000
-3.5003036E-02			
9.6940590E-02			
-1.4700000E-07	-6.1902258E-02	.00000000	.00000000
-4.3265530E-02			
-2.9940458E-03			
-4.5716691E-02	-4.9791437E-02	.00000000	.00000000
-3.5001988E-02			
-9.8792210E-02			
-7.3971062E-02	-1.8950118E-02	.00000000	.00000000
-1.3369153E-02			
-.15593016			
-7.3970534E-02	1.8951404E-02	.00000000	.00000000
1.3370060E-02			
-.15592911			
-4.5715306E-02	4.9792242E-02	.00000000	.00000000
3.5002548E-02			
-9.8789370E-02			
1.5630000E-06	6.1902273E-02	.00000000	.00000000
4.3265531E-02			
-2.9903794E-03			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 10

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	0.0
.0833	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment	ETA	CELERITY (Last Line Only)
.00000000	.00000000	6.2939282E-02	.00000000	.00000000	
4.6482220E-02	.00000000	5.0918893E-02	.00000000	9.7866442E-02	
7.5209735E-02	.00000000	1.9449131E-02	.00000000	.15835106	
7.5209557E-02	.00000000	-1.9449571E-02	.00000000	.15835068	
4.6481753E-02	.00000000	-5.0919165E-02	.00000000	9.7865453E-02	
-5.8000000E-07	.00000000	-6.2939279E-02	.00000000	-1.2217770E-06	
-4.6482692E-02	.00000000	-5.0918622E-02	.00000000	-9.7867431E-02	
-7.5209916E-02	.00000000	-1.9448692E-02	.00000000	-.15835144	
-7.5209377E-02	.00000000	1.9450009E-02	.00000000	-.15835030	
-4.6481281E-02	.00000000	5.0919437E-02	.00000000	-9.7864463E-02	
1.1600000E-06	.00000000	6.2939281E-02	.00000000	2.4447195E-06	
5.0599504					

EXPERIMENTAL RESULTS

Run Number 11

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
4" cyl.	45°	10"	.333'	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Lateral Moment ft.lbs.
0	-.066	-0.40	0.163
36	.030	0.45	-0.150
72	.155	1.15	-0.355
108	.176	1.375	-0.508
144	.086	0.900	-0.355
180	.000	0.170	-0.132
216	-.081	-0.895	0.220
252	-.130	-1.425	0.412
288	-.140	-1.50	0.510
324	-.120	-1.13	0.365
360	-.058	-0.365	0.158

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 11

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.7854
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA-CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-8.0711259E-02	1.3556066	.86440597	4.3132300E-03
.94162735			
-2.9928239E-03	5.1401008		
.94046140	1.1129530	.73598710	-.38441282
-6.6442780E-02			
9.6941615E-02	5.1401008		
1.6036812	.45788096	.30424316	-.63039264
-1.5769066E-02			
.16077232	5.1401008		
1.6542549	-.36568620	-.23855109	-.64874333
1.5287344E-02			
.16077191	5.1401008		
1.0654249	-1.0497193	-.67667498	-.41978674
5.8649902E-02			
9.6940590E-02	5.1401008		
7.1276057E-02	-1.3306928	-.86495544	-2.1183350E-02
1.0662756			
-2.9940456E-03	5.1401008		
-.94367970	-1.0945443	-.72256447	.39012173
-6.5901916E-02			
-9.8792210E-02	5.1401008		
-1.5963416	-.42636886	-.29541750	.65065809
-1.5666322E-02			
-.15593016	5.1401008		
-1.6463493	.41029211	.23059435	.65578251
1.5191465E-02			
-.15592911	5.1401008		
-1.0680311	1.0893145	.66294345	.40364841
5.8229859E-02			
-9.8789370E-02	5.1401008		
-8.0685962E-02	1.3556068	.86440703	4.3033800E-03
.94192038			
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 11

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.7854
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-4.3504400E-03	.79844474	1.2413204	-7.7979500E-03
.62065791			
-2.9928241E-03			
.92612099	.65222329	.99935320	-.37161692
.49967476			
9.6941615E-02			
1.5041883	.25060494	.38056036	-.59108048
.19027949			
.16077232			
1.5041847	-.25061060	-.38056897	-.59107913
-.19028379			
.16077192			
.92611150	-.65222668	-.99935861	-.37161330
-.49967745			
9.6940590E-02			
-4.3620200E-03	-.79844462	-1.2413204	-7.7933100E-03
-.62065795			
-2.9940458E-03			
-.92881906	-.63968263	-1.0091380	.36680138
-.50456710			
-9.8792210E-02			
-1.4971527	-.24285113	-.38660216	.60369940
-.19330037			
-.15593016			
-1.4971421	.24286761	.38662839	.60369490
.19331347			
-.15592911			
-.92879100	.63969308	1.0091539	.36678987
.50457511			
-9.8789370E-02			
-4.3272400E-03	.79844491	1.2413202	-7.8071900E-03
.62065786			
-2.9903794E-03			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 11

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.7854
.3333	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
.00000000	1.1344337	.81182070	.00000000
.00000000			
.94300269	.91777527	.65677606	-.33741583
9.7866442E-02			
1.5258088	.35055616	.25086413	-.54594971
.15835106			
1.5258050	-.35056408	-.25086978	-.54594840
.15835068			
.94299310	-.91778015	-.65677954	-.33741244
9.7865453E-02			
-1.1760000E-05	-1.1344337	-.81182068	4.2100000E-06
-1.2217770E-06			
-.94301224	-.91777036	-.65677254	.33741926
-9.7867431E-02			
-1.5258124	-.35054825	-.25085846	.54595099
-.15835144			
-1.5258013	.35057200	.25087545	.54594710
-.15835030			
-.94298360	.91778507	.65678305	.33740902
-9.7864463E-02			
2.3540000E-05	1.1344338	.81182067	-8.4300000E-06
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 12

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
Sub	45°	10"	4"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Lateral Moment ft.lbs.
0	0.0	-0.370	0.375
36	.096	0.800	0.275
72	.168	1.785	0.040
108	.160	1.740	-0.412
144	.078	0.880	-0.638
180	-.018	-0.220	-0.228
216	-.104	-1.230	-0.012
252	-.154	-1.80	0.318
288	-.162	-1.840	0.158
324	-.093	-1.230	0.323
360	0.0	-0.160	0.350

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 12

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	.7854
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force RATIO ETA-CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-.34686607	1.8130627	.48054794	-.19023335
.26385562			
-2.9928239E-03	5.1401008		
1.0711499	1.5880276	.27208392	-.36521406
-6.8259710E-02			
9.6941615E-02	5.1401008		
2.0856479	.78059646	-6.1388744E-02	-.40573238
-1.2872018E-02			
.16077232	5.1401008		
2.2835412	-.31500120	-.36519791	-.30128432
7.3860835E-03			
.16077191	5.1401008		
1.6056247	-1.2978882	-.51482938	-8.1956835E-02
3.7786663E-02			
9.6940590E-02	5.1401008		
.32566528	-1.7883219	-.47957981	.17630616
.28144983			
-2.9940456E-03	5.1401008		
-1.0831157	-1.5849077	-.26238120	.36837013
-6.6980028E-02			
-9.8792210E-02	5.1401008		
-2.0718152	-.75885390	6.6296023E-02	.41677843
-1.2742726E-02			
-.15593016	5.1401008		
-2.2630461	.36880574	.35855059	.30262819
7.3288308E-03			
-.15592911	5.1401008		
-1.6068091	1.3537978	.50590800	6.6600537E-02
3.7386962E-02			
-9.8789370E-02	5.1401008		
-.34683275	1.8130655	.48054515	-.19023872
.26388177			
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 12

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	.7854
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-.23387342	.43835437	1.6619528	-.17910207
.83097342			
-2.9928241E-03			
1.0532023	.22877252	1.4357795	-.34391059
.71788710			
9.6941615E-02			
1.9432085	-7.2679808E-02	.67136089	-.37642837
.33567919			
.16077232			
2.0844044	-.34850318	-.34047823	-.26992435
-.17023849			
.16077192			
1.4201464	-.48804073	-1.2268685	-6.4188724E-02
-.61343203			
9.6940590E-02			
.21427129	-.43707164	-1.6564970	.16843436
-.82824556			
-2.9940458E-03			
-1.0636552	-.21979586	-1.4561257	.34205610
-.72806019			
-9.8792210E-02			
-1.9300801	7.6948100E-02	-.68938506	.38595126
-.34469126			
-.15593016			
-2.0658441	.34216804	.34969621	.27766383
.17484745			
-.15592911			
-1.4218009	.47985970	1.2506016	5.9447037E-02
.62529852			
-9.8789370E-02			
-.23384252	.43835125	1.6619550	-.17910714
.83097455			
-2.9903794E-03			

POTENTIAL SOLUTION - WAVE SINE

RUN NUMBER NO. 12

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	.7854
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
-.22783398	1.5163531	.44504662	-.15880438
.00000000			
1.0761532	1.3214484	.22804316	-.31344939
9.7866442E-02			
1.9690847	.62179317	-7.6065490E-02	-.34836683
.15835106			
2.1098892	-.31536719	-.35111957	-.25021939
.15835068			
1.4447842	-1.1320673	-.49205727	-5.6496230E-02
9.7865453E-02			
.22781824	-1.5163542	-.44504497	.15880668
-1.2217770E-06			
-1.0761670	-1.3214427	-.22803989	.31345058
-9.7867431E-02			
-1.9690912	-.62178291	7.6069086E-02	.34836641
-.15835144			
-2.1098860	.31537815	.35112216	.25021757
-.15835030			
-1.4447724	1.1320748	.49205786	5.6493670E-02
-9.7864463E-02			
-.22780250	1.5163555	.44504332	-.15880901
2.4447195E-06			
5.0599504			

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER 12

WL	BL	H	DEPTH	Z	RHO	VIS	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	.7854
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force RATIO ETA	Lateral Force CELERITY	Vertical Moment	Lateral Moment
.14241650	1.8118475	.46759916	.19169182
-.64377758			
-2.9928239E-03	5.1401008		
1.4849918	1.3675264	.54521879	-5.4364837E-02
-4.0869344E-02			
9.6941615E-02	5.1401008		
2.2508878	.42337731	.39394133	-.28213441
-7.4961981E-03			
.16077232	5.1401008		
2.1403730	-.68010281	9.9991718E-02	-.41031499
1.2546915E-02			
.16077191	5.1401008		
1.2179713	-1.5334495	-.22132178	-.38208777
6.0048844E-02			
9.6940590E-02	5.1401008		
-.16389811	-1.7990193	-.46944718	-.20088372
-.55857185			
-2.9940456E-03	5.1401008		
-1.4873984	-1.3632438	-.53640372	5.9416145E-02
-4.0400702E-02			
-9.8792210E-02	5.1401008		
-2.2309538	-.38955015	-.38681404	.29754649
-7.4367725E-03			
-.15593016	5.1401008		
-2.1256296	.74113958	-.10430827	.41993030
1.2424588E-02			
-.15592911	5.1401008		
-1.2287752	1.5808456	.21154973	.37494512
5.9058263E-02			
-9.8789370E-02	5.1401008		
.14245032	1.8118447	.46760341	.19168646
-.64362417			
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER 12

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	.7854
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force AXIAL FORCE ETA	Vertical Moment	Lateral Force	Lateral Moment
.21435714 .82864214 -2.9928241E-03	.43734990	1.6572902	.16848355
1.4208121 .61372989 9.6941615E-02	.48830756	1.2274642	-6.4276127E-02
2.0853962 .17032646 .16077232	.34865410	.34065414	-.27011388
1.9441436 -.33583372 .16077192	7.2655570E-02	-.67166993	-.37665020
1.0537206 -.71822699 9.6940590E-02	-.22896111	-1.4364592	-.34408256
-.23396772 -.83137099 -2.9940458E-03	-.43863275	-1.6627479	-.17915761
-1.4224905 -.62559327 -9.8792210E-02	-.48012026	-1.2511911	5.9538064E-02
-2.0668344 -.17492537 -.15593016	-.34231067	-.34985202	.27786250
-1.9309969 .34486083 -.15592911	-7.6919930E-02	.68972420	.38617909
-1.0641510 .72841051 -9.8789370E-02	.21998577	1.4568263	.34222349
.21438818 .82864066 -2.9903794E-03	.43735326	1.6572873	.16847846

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER 12

WL S1	BL S2	H S3	DEPTH S4	Z S5	RHO S6	VISC S7	ALPHA
5.0	2.2917	.333	2.0 -.8333	1.93	9.66E-06		.7854
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
.22790985	1.5170801	.44532798	.15885454
.00000000			
1.4454616	1.1326164	.49232599	-5.6575122E-02
9.7866442E-02			
2.1108938	.31552987	.35127136	-.25039490
.15835106			
1.9700325	-.62207899	7.6042411E-02	-.34857192
.15835068			
1.0766825	-1.3220736	-.22823230	-.31360571
9.7865453E-02			
-.22792559	-1.5170789	-.44532962	-.15885223
-1.2217770E-06			
-1.4454734	-1.1326089	-.49232541	5.6577673E-02
-9.7867431E-02			
-2.1108971	-.31551891	-.35126876	.25039674
-.15835144			
-1.9700261	.62208922	-7.6038782E-02	.34857232
-.15835030			
-1.0766688	1.3220793	.22823556	.31360453
-9.7864463E-02			
.22794135	1.5170776	.44533126	.15884992
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 13

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
2-7/8" cyl.	45°	10"	4"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Lateral Moment ft.lbs.
0	-.060	-0.175	0.062
36	.050	0.220	-0.070
72	.158	0.500	-0.158
108	.171	0.550	-0.222
144	.087	0.340	-0.128
180	-.020	-0.025	-0.015
216	-.045	-0.430	0.095
252	-.148	-0.680	0.251
288	-.151	-0.730	0.248
324	-.125	-0.525	0.190
360	-.060	-0.175	0.060

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 13

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.7854
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment	RATIO ETA-CELERITY
-5.4521970E-02	.69850443	.44399876	4.4500330E-03	.95561037
-2.9928239E-03	5.1401008			
.47238862	.57711209	.38302166	-.19452101	
-9.0653981E-02				
9.6941615E-02	5.1401008			
.81948759	.24304751	.16200587	-.32232331	
-2.1124161E-02				
.16077232	5.1401008			
.85410791	-.17999241	-.11682648	-.33472755	
2.0268351E-02				
.16077191	5.1401008			
.55803130	-.53378292	-.34266005	-.21857038	
7.6741283E-02				
9.6940590E-02	5.1401008			
4.9675167E-02	-.68136093	-.44437083	-1.3115675E-02	1.0488501
-2.9940456E-03	5.1401008			
-.47409111	-.56271369	-.37615274	.19911275	
-8.9903309E-02				
-9.8792210E-02	5.1401008			
-.81576235	-.22052879	-.15740188	.33371347	
-2.0989261E-02				
-.15593016	5.1401008			
-.85000207	.20923694	.11280934	.33736293	
2.0145119E-02				
-.15592911	5.1401008			
-.55932092	.55906414	.33558003	.20862138	
7.6204914E-02				
-9.8789370E-02	5.1401008			
-5.4509106E-02	.69850469	.44399947	4.4450320E-03	.95583692
-2.9903792E-03	5.1401008			

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 13

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.7854
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment	AXIAL FORCE	ETA
-2.2478100E-03	.41254883	.64137845	-4.0291280E-03	.32068804	-2.9928241E-03
.47851791	.33699758	.51635633	-.19201093	.25817721	9.6941615E-02
.77719991	.12948521	.19663193	-.30540568	9.8315610E-02	.16077232
.77719801	-.12948814	-.19663636	-.30540498	-9.8317831E-02	.16077192
.47851305	-.33699935	-.51635911	-.19200906	-.25817861	9.6940590E-02
-2.2538100E-03	-.41254875	-.64137851	-4.0267310E-03	-.32068807	-2.9940458E-03
-.47991197	-.33051795	-.52141198	.18952277	-.26070504	-9.8792210E-02
-.77356464	-.12547887	-.19975367	.31192573	- 9.9876470E-02	-.15593016
-.77355916	.12548739	.19976721	.31192343	9.9883238E-02	-.15592911
-.47989755	.33052333	.52142027	.18951685	.26070917	-9.8789370E-02
-2.2358300E-03	.41254890	.64137838	-4.0338920E-03	.32068802	-2.9903794E-03

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 13

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	.7854
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
.00000000	.58615118	.41946006	7.0000000E-09
.00000000			
.48724054	.47420577	.33934995	-.17433956
9.7866442E-02			
.78837093	.18112906	.12961911	-.28208701
.15835106			
.78836904	-.18113314	-.12962204	-.28208636
.15835068			
.48723562	-.47420830	-.33935175	-.17433780
9.7865453E-02			
-6.0600000E-06	-.58615120	-.41946005	2.1830000E-06
-1.2217770E-06			
-.48724547	-.47420324	-.33934814	.17434131
-9.7867431E-02			
-.78837280	-.18112495	-.12961618	.28208769
-.15835144			
-.78836716	.18113723	.12962497	.28208569
-.15835030			
-.48723070	.47421085	.33935357	.17433604
-9.7864463E-02			
1.2150000E-05	.58615118	.41946002	-4.3530000E-06
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 14

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
Sub	90°	10"	4"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Lateral Force lbs.	Lateral Moment ft.lbs.
0	-.040	-0.000	2.970	0.668
36	.018	1.125	2.460	0.430
72	.167	1.875	0.524	0.150
108	.180	1.680	-1.335	-0.350
144	.090	0.600	-2.740	-0.525
180	.006	-0.525	-2.810	-0.574
216	-.082	-1.400	-1.950	-0.318
252	-.128	-1.875	-0.581	-0.158
288	-.150	-1.825	0.647	0.173
324	-.130	-1.280	2.050	0.380
360	-.082	-0.100	2.900	0.605

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 14

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	1.5708
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force RATIO ETA-CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-.13035610	2.7810507	1.1714574E-02	-.32546625
.88040191			
-2.9928239E-03	5.1401008		
1.3976820	2.2825804	-.16533858	-.26537561
-5.2303486E-02			
9.6941615E-02	5.1401008		
2.3905812	.97067518	-.27920571	-.10979980
-4.0181787E-03			
.16077232	5.1401008		
2.4097864	-.71302742	-.28079203	8.7388972E-02
3.9863515E-03			
.16077191	5.1401008		
1.5438726	-2.1558606	-.17781536	.25443751
4.7351346E-02			
9.6940590E-02	5.1401008		
9.9154320E-02	-2.7660623	-8.0747160E-03	.32419009
1.1574458			
-2.9940456E-03	5.1401008		
-1.4084771	-2.2724493	.16656457	.26599728
-5.1089904E-02			
-9.8792210E-02	5.1401008		
-2.3655876	-.89879094	.27628350	.10439476
-3.9639019E-03			
-.15593016	5.1401008		
-2.3843253	.84581929	.27782829	-9.9927040E-02
3.9333142E-03			
-.15592911	5.1401008		
-1.5523515	2.2645414	.17883774	-.26535734
4.6356493E-02			
-9.8789370E-02	5.1401008		
-.13031927	2.7810505	1.1710319E-02	-.32546621
.88065072			
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 14

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	1.5708
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment	AXIAL FORCE	ETA
-1.4343673E-02	1.6637308E-03	2.5696509	-.29846216	-4.7140246E-06	-2.9928241E-03
1.3531741	-.15717159	2.0560618	-.23880787	-3.7718456E-06	9.6941615E-02
2.2082557	-.25648726	.77995005	-9.0588050E-02	-1.4308182E-06	.16077232
2.2082495	-.25648513	-.77997022	9.0595010E-02	1.4308553E-06	.16077192
1.3531582	-.15716606	-2.0560748	.23881233	3.7718689E-06	9.6940590E-02
-1.4363073E-02	1.6704736E-03	-2.5696513	.29846226	4.7140252E-06	-2.9940458E-03
-1.3620530	.15820277	-2.1017036	.24410920	3.8555752E-06	-9.8792210E-02
-2.1850503	.25379193	-.80814642	9.3862940E-02	1.4825446E-06	-.15593016
-2.1850342	.25378870	.80820372	-9.3874370E-02	-1.4826496E-06	-.15592911
-1.3620105	.15819431	2.1017380	-.24411604	-3.8556383E-06	-9.8789370E-02
-1.4309783E-02	1.6598486E-03	2.5696501	-.29846208	4.7140232E-06	-2.9903794E-03

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 14

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.3333	2.0	-.8333	1.93	9.66E-06	1.5708
.4725	-.1319	-.1529	.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
1.2587347E-06	2.3505011	-2.4579117E-06	-.27300818
.00000000			
1.3815936	1.9015927	-.16047248	-.22086658
9.7954610E-02			
2.2354619	.72633710	-.25964720	-8.4361112E-02
.15849372			
2.2354559	-.72635584	-.25964510	8.4367592E-02
.15849334			
1.3815778	-1.9016042	-.16046694	.22087059
9.7953620E-02			
-1.8483919E-05	-2.3505011	4.4042005E-06	.27300818
-1.2228777E-06			
-1.3816076	-1.9015823	.16047406	.22086543
-9.7955600E-02			
-2.2354672	-.72632068	.25964781	8.4359212E-02
-.15849410			
-2.2354505	.72637227	.25964445	-8.4369492E-02
-.15849296			
-1.3815637	1.9016144	.16046531	-.22087179
-9.7952629E-02			
3.5747789E-05	2.3505011	-6.4088771E-06	-.27300818
2.4469220E-06			
5.0599504			

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER 14

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	1.5708
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force RATIO ETA	Lateral Force CELERITY	Vertical Moment	Lateral Moment
-.13040307	2.7824348	-1.1721280E-02	.32557516
.88036485			
-2.9928239E-03	5.1401008		
1.3983871	2.2837083	.16539219	.26546633
-5.2293609E-02			
9.6941615E-02	5.1401008		
2.3917694	.97113828	.27929969	.10983879
-4.0173267E-03			
.16077232	5.1401008		
2.4109818	-.71340309	.28088945	-8.7415700E-02
3.9854807E-03			
.16077191	5.1401008		
1.5446282	-2.1569441	.17787858	-.25452046
4.7343124E-02			
9.6940590E-02	5.1401008		
9.9185720E-02	-2.7674421	8.0800517E-03	-.32429866
1.1574476			
-2.9940456E-03	5.1401008		
-1.4091875	-2.2735824	-.16661884	-.26608952
-5.1080306E-02			
-9.8792210E-02	5.1401008		
-2.3667636	-.89923661	-.27637657	-.10443455
-3.9630652E-03			
-.15593016	5.1401008		
-2.3855079	.84624299	-.27792466	9.9955973E-02
3.9324517E-03			
-.15592911	5.1401008		
-1.5531113	2.2656690	-.17890115	.26544327
4.6348326E-02			
-9.8789370E-02	5.1401008		
-.13036613	2.7824346	-1.1716937E-02	.32557516
.88061415			
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER 14

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	1.5708
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-1.4353223E-02	-1.6691295E-03	2.5709183	.29856379
-4.7163498E-06			
-2.9928241E-03			
1.3538396	.15722112	2.0570776	.23889235
-3.7737089E-06			
9.6941615E-02			
2.2093442	.25657303	.78033725	9.0623791E-02
-1.4315287E-06			
.16077232			
2.2093395	.25657389	-.78035240	-9.0620931E-02
1.4315564E-06			
.16077192			
1.3538276	.15722341	-2.0570872	-.23889047
3.7737266E-06			
9.6940590E-02			
-1.4367740E-02	-1.6663234E-03	-2.5709186	-.29856376
4.7163503E-06			
-2.9940458E-03			
-1.3627229	-.15825284	-2.1027417	-.24419526
3.8574796E-06			
-9.8792210E-02			
-2.1861273	-.25387688	-.80854764	-9.3899964E-02
1.4832805E-06			
-.15593016			
-2.1861126	-.25387652	.80859976	9.3901248E-02
-1.4833762E-06			
-.15592911			
-1.3626843	-.15825189	2.1027730	.24419606
-3.8575372E-06			
-9.8789370E-02			
-1.4319258E-02	-1.6651312E-03	2.5709177	.29856371
-4.7163486E-06			
-2.9903794E-03			

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER 14

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.2917	.333	2.0	-.8333	1.93	9.66E-06	1.5708
.4726	.1319	-.1529	-.0854	-.1529	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			
-1.2580297E-06	2.3495438	-2.4572832E-06	.27285526
.00000000			
1.3810288	1.9008195	.16037862	.22074575
9.7866442E-02			
2.2345507	.72604359	.25950027	8.4318433E-02
.15835106			
2.2345462	-.72605764	.25950115	-8.4315783E-02
.15835068			
1.3810170	-1.9008282	.16038093	-.22074405
9.7865453E-02			
-1.6018482E-05	-2.3495438	3.9613342E-07	-.27285526
-1.2217770E-06			
-1.3810429	-1.9008094	-.16038031	-.22074454
-9.7867431E-02			
-2.2345560	-.72602716	-.25950091	-8.4316529E-02
-.15835144			
-2.2345408	.72607407	-.25950054	8.4317687E-02
-.15835030			
-1.3810030	1.9008383	-.16037932	.22074525
-9.7864463E-02			
3.3275712E-05	2.3495438	1.6081609E-06	.27285526
2.4447195E-06			
5.0599504			

EXPERIMENTAL RESULTS

Run Number 15

Model	Angle of Attack	Depth Below Surface	Wave Height	Wave Length	Depth of Water
2-7/8" cyl.	90°	10"	4"	5.0'	2.0'

Phase Angle	ETA ft.	Vertical Force lbs.	Lateral Force lbs.
0	-.04	-0.280	1.344
36	.018	0.175	1.725
72	.167	0.480	1.270
108	.180	0.530	0.000
144	.090	0.345	-1.190
180	-.006	0.005	-1.672
216	-.082	-0.345	-1.780
252	-.128	-0.610	-1.225
288	-.150	-0.805	-0.291
324	-.130	-0.620	0.618
360	-.082	-0.300	1.620

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 15

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	1.5708
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA-CELERITY	Lateral Force	Vertical Moment	Lateral Moment
-8.4924284E-02	1.2068576	-2.6120000E-06	-6.0000000E-08
.92041975			
-2.9928239E-03	5.1401008		
.58738901	1.0044417	-2.3280000E-06	1.5200000E-06
-8.5342747E-02			
9.6941615E-02	5.1401008		
1.0333142	.45211919	-9.1000000E-07	2.5700000E-06
-6.8303471E-03			
.16077232	5.1401008		
1.0474277	-.27722200	7.1000000E-07	2.7860000E-06
6.7386139E-03			
.16077191	5.1401008		
.68764178	-.91818883	1.9400000E-06	1.8800000E-06
7.2901237E-02			
9.6940590E-02	5.1401008		
7.1399702E-02	-1.1960239	2.5630000E-06	1.1000000E-07
1.0947662			
-2.9940456E-03	5.1401008		
-.59234245	-.98560006	2.2090000E-06	-1.6100000E-06
-8.3328616E-02			
-9.8792210E-02	5.1401008		
-1.0225424	-.39617175	9.1000000E-07	-2.6930000E-06
-6.7448044E-03			
-.15593016	5.1401008		
-1.0363297	.35955338	-6.7000000E-07	-2.8250000E-06
6.6559595E-03			
-.15592911	5.1401008		
-.69104321	.97972001	-1.8400000E-06	-1.7300000E-06
7.1429343E-02			
-9.8789370E-02	5.1401008		
-8.4908318E-02	1.2068577	-2.5610000E-06	-1.0000000E-08
.92059300			
-2.9903792E-03	5.1401008		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER NO. 15

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	-.8333	1.93	9.66E-06	1.5708
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment	AXIAL FORCE	ETA
-6.2769549E-03	-2.4270400E-06	1.1244115	4.0000000E-08	-2.0627328E-06	-2.9928241E-03
.59211291	-1.9860000E-06	.89967887	1.6000000E-06	-1.6504608E-06	9.6941615E-02
.96627438	-7.5000000E-07	.34128612	2.4880000E-06	-6.2608937E-07	.16077232
.96627204	7.5000000E-07	-.34129384	2.4760000E-06	6.2610355E-07	.16077192
.59210682	1.9520000E-06	-.89968374	1.5800000E-06	1.6504697E-06	9.6940590E-02
-6.2843737E-03	2.3788500E-06	-1.1244116	4.0000000E-08	2.0627330E-06	-2.9940458E-03
-.59599813	1.9000000E-06	-.91965041	-1.5300000E-06	1.6870986E-06	-9.8792210E-02
-.95612028	7.0000000E-07	-.35362413	-2.5540000E-06	6.4872346E-07	-.15593016
-.95611358	-7.1000000E-07	.35364806	-2.5550000E-06	-6.4876735E-07	-.15592911
-.59598038	-1.9070000E-06	.91966481	-1.5500000E-06	-1.6871250E-06	-9.8789370E-02
-6.2621124E-03	-2.3788400E-06	1.1244112	5.0000000E-08	-2.0627322E-06	-2.9903794E-03

POTENTIAL SOLUTION - SINE WAVE

RUN NUMBER NO. 15

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
5.0	2.5	.333	2.0	.8333	1.93	9.66E-06	1.5708
.23958	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
ETA			
CELERITY (Last Line Only)			

2.0000000E-13	8.3439272	-2.0038592E-05	.00000000
.00000000			
4.9044471	6.7503716	-1.6060000E-05	1.1700000E-05
9.7866442E-02			
7.9355537	2.5783919	-6.0000000E-06	1.8690000E-05
.15835106			
7.9355348	-2.5784501	6.1000000E-06	1.8690000E-05
.15835068			
4.9043975	-6.7504074	1.5920000E-05	1.1500000E-05
9.7865453E-02			
-6.1252766E-05	-.3439272	1.9640178E-05	.00000000
-1.2217770E-06			
-4.9044966	-6.7503354	1.5980000E-05	-1.1700000E-05
-9.7867431E-02			
-7.9355726	-2.5783335	6.0000000E-06	-1.8690000E-05
-.15835144			
-7.9355159	2.5785082	-6.1000000E-06	-1.8690000E-05
-.15835030			
-4.9043480	6.7504436	-1.6010000E-05	-1.1500000E-05
-9.7864463E-02			
1.2253890E-04	8.3439272	-1.9640177E-05	.00000000
2.4447195E-06			
5.0599504			

APPENDIX B. PROTOTYPE RUNS - THEORY ONLY

This appendix contains the tables of results obtained by each of the three theories for the two prototype bodies investigated. These tables should be interpreted in the same manner as the tables in Appendix A.

The tables included are:

Run No.	Body	Angle of Attack	Wave Length	Wave Height
P1	Sub	0°	400	25
P2	Sub	45°	400	25
P3	Sub	90°	400	25
P4	3 ft. Cyl.	0°	400	25
P5	3 ft. Cyl.	45°	400	25
P6	3 ft. Cyl.	90°	400	25

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER P1

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
400.	250.	25.	900.	-80.	1.93	9.66E-06	.00001

S1	S2	S3	S4
31.1558	4.0329E-02	-1.6597E-03	3.0559E-06

S5	S6	S7
8.8705E-08	-5.1805E-10	-7.3899E-12

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
RATIO			
ETA	CELERITY		
74538.300	6.5026223	29896485.	36.580903
-.10683011			
-1.7792336E-06	46.121821		
435068.94	4.7691744	26619339.	-150.52323
-1.4848854E-02			
7.3473300	46.121821		
629486.25	1.2140561	13171736.	-280.13237
3.8461091E-03			
11.888219	46.121821		
584478.32	-2.8047922	-5288808.4	-302.74000
6.0928432E-03			
11.888191	46.121821		
315352.20	-5.7522996	-21737195.	-209.71068
2.3201865E-02			
7.3472560	46.121821		
-74542.960	-6.5026138	-29896515.	-36.578644
-.10682333			
-9.3504230E-05	46.121821		
-435072.43	-4.7691390	-26619236.	150.52526
-1.4848660E-02			
-7.3474055	46.121821		
-629487.17	-1.2140064	-13171538.	280.13332
-3.8459941E-03			
-11.888245	46.121821		
-584476.37	2.8048376	5289027.2	302.73953
6.0929667E-03			
-11.888160	46.121821		
-315348.08	5.7523250	21737350.	209.70899
2.3202257E-02			
-7.3471825	46.121821		
74547.674	6.5026104	29896545.	36.576416
-.10681649			
1.8175827E-04	46.121821		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER P1

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
400.	250.	25.	900.	-80.	1.93	9.66E-06	0.0

S1	S2	S3	S4
31.1558	4.0329E-02	-1.6597E-03	3.0559E-06

S5	S6	S7
8.8705E-08	-5.1805E-10	-7.3899E-12

FORMAT

Vertical Force AXIAL FORCE ETA	Vertical Moment	Lateral Force	Lateral Moment
75340.444 292910.92 -1.7792336E-06	27606684.	.00000000	.00000000
405289.76 214827.62 7.3473300	24271339.	.00000000	.00000000
580431.46 54687.091 11.888219	11665127.	.00000000	.00000000
533867.10 -126342.14 11.888191	-5396787.6	.00000000	.00000000
283382.68 -259112.74 7.3472560	-20397303.	.00000000	.00000000
-75344.747 -292910.64 -9.3504230E-05	-27606708.	.00000000	.00000000
-405292.94 -214826.14 -7.3474055	-24271240.	.00000000	.00000000
-580432.26 -54684.958 -11.888245	-11664942.	.00000000	.00000000
-533865.24 126344.10 -11.888160	5396988.8	.00000000	.00000000
-283378.88 259113.78 -7.3471825	20397443.	.00000000	.00000000
75349.048 292910.36 1.8175827E-04	27606732.	.00000000	.00000000

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER P2

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
400.	250.	25.	900.	-80.	1.93	9.66E-06	.785

S1	S2	S3	S4
31.1558	4.0329E-02	-1.6597E-03	3.0559E-06

S5	S6	S7
8.8705E-08	-5.1805E-10	-7.3899E-12

FORMAT

Vertical Force	Lateral Force	Vertical Moment	Lateral Moment
RATIO ETA	CELERITY		
52398.946	540860.11	23791644.	3519782.3
-.20312320			
-1.7792336E-06	46.121821		
490911.25	413640.46	22396650.	-7004072.0
-1.6581949E-02			
7.3473300	46.121821		
741381.64	129083.47	12302205.	-14901050.
-4.1239994E-03			
11.888219	46.121821		
709285.35	-204576.22	-2392214.1	-17150768.
5.3722337E-03			
11.888191	46.121821		
406039.31	-459754.37	-16094864.	-12801209.
2.2158463E-02			
7.3472560	46.121821		
-52404.544	-538490.31	-23791683.	-3509499.7
-.20310139			
-9.3504230E-05	46.121821		
-490915.57	-410808.17	-22396577.	7092794.7
-1.6581720E-02			
-7.3474055	46.121821		
-741382.99	-125447.79	-12302050.	14959752.
-4.1238890E-03			
-11.888245	46.121821		
-709283.28	208106.34	2392391.6	17116605.
5.3723510E-03			
-11.888160	46.121821		
-406034.55	462362.04	16094997.	12731170.
2.2158834E-02			
-7.3471825	46.121821		
52410.149	540859.52	23791723.	3519536.3
-.20307938			
1.8175827E-04	46.121821		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER P2

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
400.	250.	25.	900.	-80.	1.93	9.66E-06	.785

S1	S2	S3	S4
31.1558	4.0329E-02	-1.6597E-03	3.0559E-06

S5	S6	S7
8.8705E-08	-5.1805E-10	-7.3899E-12

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
57572.878	21999681.	495399.38	3102645.1
247897.01			
-1.7792336E-06			
458544.91	20378215.	376866.73	-6629946.5
188583.47			
7.3473300			
684367.62	10972928.	114383.16	-13830112.
57237.141			
11.888219			
648783.94	-2623661.3	-191791.09	-15747622.
-95971.949			
11.888191			
365385.75	-15218097.	-424707.31	-11650047.
-212522.83			
7.3472560			
-57578.005	-21999712.	-495399.08	-3102531.0
-247896.87			
-9.3504230E-05			
-458548.87	-20378146.	-376864.37	6630053.3
-188582.28			
-7.3474055			
-684368.79	-10972785.	-114379.60	13830169.
-57235.360			
-11.888245			
-648781.97	2623826.5	191794.46	15747607.
95973.633			
-11.888160			
-365381.34	15218218.	424709.23	11649969.
212523.79			
-7.3471825			
57583.162	21999744.	495398.79	3102417.2
247896.72			
1.8175827E-04			

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER P3

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
400.	250.	25.	900.	-80.	1.93	9.66E-06	1.5708

S1	S2	S3	S4
31.1558	4.0329E-02	-1.6597E-03	3.0559E-06

S5	S6	S7
8.8705E-08	-5.1805E-10	-7.3899E-12

FORMAT

Vertical Force RATIO ETA	Lateral Force CELERITY	Vertical Moment	Lateral Moment
-15837.002 .99997777	899610.69	-56209.530	6099689.8
-1.7792336E-06 520359.89	46.121821 733516.35	3559510.8	4955091.8
-1.9695024E-02 7.3473300	46.121821		
857165.51 -1.6057055E-03	290451.46	5813676.2	1928641.7
11.888219 859916.44	46.121821 -265169.60	5824150.7	-1839775.3
1.6006424E-03 11.888191	46.121821		
540852.19 1.8949010E-02	-721117.47	3633037.0	-4910747.2
7.3472560	46.121821		
15830.375 1.0003963	-898412.64	56163.450	-6095139.5
-9.3504230E-05	46.121821		
-520365.37 -1.9694590E-02	-727996.19	-3559548.0	-4935552.2
-7.3474055	46.121821		
-857167.63 -1.6056246E-03	-281360.54	-5813690.8	-1897139.8
-11.888245	46.121821		
-859914.46 1.6007214E-03	274260.50	-5824137.9	1871270.9
-11.888160	46.121821		
-540846.97 1.8949391E-02	726637.61	-3633000.5	4930283.7
-7.3471825	46.121821		
-15823.744 1.0008156	899610.77	-56118.520	6099692.3
1.8175827E-04	46.121821		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER P3

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
400.	250.	25.	900.	-80.	1.93	9.66E-06	1.5708

S1	S2	S3	S4
31.1558	4.0329E-02	-1.6597E-03	3.0559E-06

S5	S6	S7
8.8705E-08	-5.1805E-10	-7.3899E-12

FORMAT

Vertical Force AXIAL FORCE ETA	Vertical Moment	Lateral Force	Lateral Moment
-.32203813	-128.36620	830610.71	5587767.5
-1.5237554			
-1.7792336E-06			
488221.46	3284310.1	671977.65	4520668.8
-1.2327430			
7.3473300			
789958.39	5314249.5	256670.79	1726820.2
-.47086261			
11.888219			
789956.70	5314313.9	-256675.99	-1726617.0
.47087210			
11.888191			
488217.03	3284482.9	-671980.85	-4520543.9
1.2327488			
7.3472560			
-5.7766547	86.243699	-830610.71	-5587767.5
1.5237554			
-9.3504230E-05			
-488226.40	-3284343.9	-671974.07	-4520644.0
1.2327364			
-7.3474055			
-789960.25	-5314261.6	-256665.00	-1726781.5
.47085194			
-11.888245			
-789954.82	-5314301.8	256681.80	1726656.5
-.47088275			
-11.888160			
-488212.10	-3284450.3	671984.46	4520568.2
-1.2327554			
-7.3471825			
11.877490	-45.201560	830610.71	5587767.5
-1.5237554			
1.8175827E-04			

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER P4

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
400.	30.	25.	200.	-50.	1.93	9.66E-06	.00001
3.0	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA	Lateral Force CELERITY	Vertical Moment	Lateral Moment
-532.35097	2.7130823E-02	1138.6820	-7.5117000E-05
.99641165			
-.10785793	46.045438		
226.05225	2.1919489E-02	1547.5368	-1.8881820E-02
-1.5069499			
7.3140010	46.045438		
847.39942	8.3656598E-03	608.97260	-3.0453136E-02
-8.5016241E-02			
11.975477	46.045438		
991.48486	-8.3650646E-03	-101.79570	-3.0453407E-02
7.2663654E-02			
11.975448	46.045438		
907.34958	-2.1919131E-02	-307.91550	-1.8882354E-02
.37543818			
7.3139255	46.045438		
528.52299	-2.7130696E-02	-1144.0439	-7.5401000E-05
1.0036277			
-.10794965	46.045438		
-228.91758	-2.1979087E-02	-1538.5595	1.8835693E-02
-1.4807528			
-7.3807330	46.045438		
-844.70425	-8.4021924E-03	-601.61430	3.0575267E-02
-8.4821649E-02			
-11.800986	46.045438		
-988.00271	8.4026244E-03	98.407200	3.0575219E-02
7.2525922E-02			
-11.800904	46.045438		
-906.85234	2.1979415E-02	299.51630	1.8835313E-02
.37380010			
-7.3805145	46.045438		
-532.33647	2.7130821E-02	1138.6967	-7.5597000E-05
.99643817			
-.10767439	46.045438		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER P4

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
400.	30.	25.	200.	-50.	1.93	9.66E-06	0.0
3.0	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force AXIAL FORCE ETA	Vertical Moment	Lateral Force	Lateral Moment
-4.7516730 1222.1132 -.10785793	2839.0509	.00000000	.00000000
1409.6328 987.35929 7.3140010	2307.6479	.00000000	.00000000
2287.0513 376.81598 11.975477	883.98800	.00000000	.00000000
2287.0459 -376.82450 11.975448	-884.00800	.00000000	.00000000
1409.6185 -987.36455 7.3139255	-2307.6598	.00000000	.00000000
-4.7692880 -1222.1132 -.10794965	-2839.0505	.00000000	.00000000
-1412.5836 -990.05408 -7.3807330	-2286.0161	.00000000	.00000000
-2279.3685 -378.47613 -11.800986	-870.60640	.00000000	.00000000
-2279.3522 378.50180 -11.800904	870.66540	.00000000	.00000000
-1412.5410 990.06990 -7.3805145	2286.0527	.00000000	.00000000
-4.7163930 1222.1131 -.10767439	2839.0510	.00000000	.00000000

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER P5

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
400.	30.	25.	200.	-50.	1.93	9.66E-06	.785
3.0	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA	Lateral Force CELERITY	Vertical Moment	Lateral Moment
-537.66529	719.53697	807.69670	49.124700
.99638005			
-.10785793	46.045438		
227.29197	698.46102	1101.9172	-124.17170
-1.5048250			
7.3140010	46.045438		
849.16458	493.76305	411.36610	-426.09400
-8.7818059E-02			
11.975477	46.045438		
998.30831	75.382443	-92.855200	-649.32555
7.4700303E-02			
11.975448	46.045438		
911.35985	-397.81182	-214.30870	-542.64049
.37530623			
7.3139255	46.045438		
533.76537	-637.40598	-811.56440	-51.804800
1.0036597			
-.10794965	46.045438		
-230.22869	-559.46373	-1095.4742	266.21960
-1.4781238			
-7.3807330	46.045438		
-846.33640	-256.11165	-406.59090	522.52789
-8.7734782E-02			
-11.800986	46.045438		
-994.84164	164.14775	89.948200	557.23478
7.4643684E-02			
-11.800904	46.045438		
-910.83630	539.84917	208.32270	398.93070
.37363278			
-7.3805145	46.045438		
-537.65125	719.53842	807.70720	49.118100
.99640642			
-.10767439	46.045438		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER P5

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
400.	30.	25.	200.	-50.	1.93	9.66E-06	.785
3.0	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force AXIAL FORCE ETA	Vertical Moment	Lateral Force	Lateral Moment
-4.8413650 868.52576 -.10785793	2013.8965	1735.6688	-3.4335000
1416.1622 701.67832 7.3140010	1637.0074	1402.2395	-852.92800
2297.7335 267.78543 11.975477	627.10130	535.14457	-1375.5690
2297.7279 -267.79148 11.975448	-627.11540	-535.15664	-1375.5659
1416.1478 -701.68207 7.3139255	-1637.0154	-1402.2470	-852.91890
-4.8590710 -868.52575 -.10794965	-2013.8952	-1735.6687	-3.4230000
-1419.1686 -703.62060 -7.3807330	-1621.5329	-1406.1210	850.81360
-2289.9054 -268.98205 -11.800986	-617.52880	-537.53592	1381.1282
-2289.8892 269.00030 -11.800904	617.57130	537.57235	1381.1181
-1419.1257 703.63184 -7.3805145	1621.5587	1406.1434	850.78790
-4.8059270 868.52572 -.10767439	2013.8955	1735.6686	-3.4546000

MODIFIED MORRISON SOLUTION - STOKIAN WAVE

RUN NUMBER P6

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
400.	30.	25.	200.	-50.	1.93	9.66E-06	1.5708
3.0	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force RATIO ETA	Lateral Force CELERITY	Vertical Moment	Lateral Moment
-543.05184 .99634869	1070.6491	-4.2000000E-03	-1.1000000E-03
-.10785793 228.52365	46.045438 1140.3940	-2.7000000E-03	2.0000000E-04
-1.5028611 7.3140010	46.045438		
848.27305 -9.4046414E-02	914.79429	-2.9000000E-03	2.8000000E-03
11.975477 1007.8263	46.045438 286.66058	1.0000000E-04	4.6700000E-03
7.9158946E-02 11.975448	46.045438		
915.40059 .37518376	-505.51289	1.0000000E-03	4.1000000E-03
7.3139255 539.07899	46.045438 -966.65891	3.7000000E-03	8.0000000E-04
1.0036914 -.10794965	46.045438		
-231.53307 -1.4756470	-840.77703	5.0400000E-03	-1.8000000E-03
-7.3807330 -845.33244	46.045438 -379.68211	9.0000000E-04	-3.7800000E-03
-9.4059494E-02 -11.800986	46.045438		
-1004.3534 7.9171370E-02	251.31371	4.0000000E-04	-3.3400000E-03
-11.800904 -914.84950	46.045438 809.75845	-4.0000000E-03	-2.1000000E-03
.37347413 -7.3805145	46.045438		
-543.03754 .99637494	1070.6527	-3.7000000E-03	-1.0000000E-03
-.10767439	46.045438		

POTENTIAL SOLUTION - STOKIAN WAVE

RUN NUMBER P6

WL	BL	H	DEPTH	Z	RHO	VISC	ALPHA
S1	S2	S3	S4	S5	S6	S7	
400.	30.	25.	200.	-50.	1.93	9.66E-06	1.5708
3.0	0.0	0.0	0.0	0.0	0.0	0.0	

FORMAT

Vertical Force	Vertical Moment	Lateral Force	Lateral Moment
AXIAL FORCE			
ETA			
-4.9322170	-1.0474700E-02	2466.9918	.00000000
-4.5256967E-03			
-.10785793			
1422.7199	-8.7000000E-03	1993.0335	6.3000000E-03
-3.6562199E-03			
7.3140010			
2308.4630	-2.9000000E-03	760.60336	8.9000000E-03
-1.3953268E-03			
11.975477			
2308.4575	2.8000000E-03	-760.62053	9.0000000E-03
1.3953583E-03			
11.975448			
1422.7054	7.6000000E-03	-1993.0441	5.9000000E-03
3.6562394E-03			
7.3139255			
-4.9500041	9.2906000E-03	-2466.9918	.00000000
4.5256967E-03			
-.10794965			
-1425.7825	7.7000000E-03	-1998.6281	-5.5000000E-03
3.6664833E-03			
-7.3807330			
-2300.4881	2.6000000E-03	-764.05036	-9.0000000E-03
1.4016504E-03			
-11.800986			
-2300.4716	-2.8000000E-03	764.10214	-8.8000000E-03
-1.4017454E-03			
-11.800904			
-1425.7393	-7.7000000E-03	1998.6600	-5.9000000E-03
-3.6665419E-03			
-7.3805145			
-4.8966323	-9.2877000E-03	2466.9918	.00000000
-4.5256967E-03			
-.10767439			

APPENDIX C. COMPUTER PROGRAMS

This section contains the IBM 1620 computer programs utilized in the calculations for each of the three theories discussed. The programs are written in the standard IBM Fortran language as adapted for the IBM 1620 computer.

As an aid to the reader in the event that a close inspection of the mechanics of a program is desired, a list and description of the symbols used in the modified Morrison theory is given below. The explanation does not cover every symbol but does cover the most used symbols and those which might be difficult to understand. Although the Stokian potential flow program is not explained, its structure is similar to the modified Morrison program. The sine wave potential program simply involved the programming of equations derived directly from Cummins (6) report, and for this reason the symbols involved are not discussed.

Program Symbol	Meaning in Theory Symbols (Explanation)
WL	L (wavelength)
BL	ℓ (body length)
H	H (wave height-double amplitude)
Depth	h (depth of water)
z	z (depth of the body axis below still water)

Program Symbol	Meaning in Theory Symbols (Explanation)
RHO	ρ_f (fluid density)
VISC	ν (fluid viscosity)
ALPHA	α (angle of attack)
S1, S2 ... S7	Coefficients of a 6th order parabola ($S1+S2x+S3x^2$ ... $S7x^6$) describing the variation of body diameter over the length. x=0 is located at midships, +x towards the bow
B	K (wavenumber = $2\pi/L$)
CEL	C (wave celerity)
SIGMA	$\sigma = \frac{2\pi c}{L}$ (wave angular frequency)
TIME	T (wave period)
DIAM	Part of 6th order parabola describing the diameter
D(J)	D(x) Diameter of the body at a particular axial position
X(J)	Axial position along the axis of symmetry. Midship section is the origin. x(1), x(2) ... x(7) locate the seven Tchebycheff stations.
PIHOL	$\frac{\pi H}{L}$
PIHSQ	$(\frac{\pi H}{L})^2$
ALSIN	$\sin \alpha$
ALCOS	$\cos \alpha$
Y	$\psi = (Kx \cos\alpha + Ky \sin\alpha + \sigma t)$ Note that y = 0 for all cases
T	Stepping parameter simulating the passage of time. It was stepped from 0 to 10 inclusive giving eleven sets of forces and moments over the wave cycle.

Program Symbol	Meaning in Theory Symbols (Explanation)
HH(1)	v_{max}
HH(2)	w_{max}
CMCN	$\frac{u_{max}T}{D(x)}$ or $\frac{w_{max}T}{D(x)}$ (depending on index L)
CM(J,L)	C_m (vertical and transverse for each x(J) at a particular time)
RNCON	$\frac{\lvert v\rvert D(x)}{\nu}$ or $\frac{\lvert w\rvert D}{\nu}$ (depending on index K)
CD(K,K)	C_D (corresponds to vertical and transverse Reynolds number at each x(J))
ETA	η (displacement of free surface from still water)
VFI	ΔF_{I_z} (incremental vertical component of vertical force)
VFD	ΔF_{D_z} (incremental drag component of vertical force)
HFI	ΔF_{I_y} (incremental inertial component of transverse force)
HFD	ΔF_{D_y} (incremental drag component of transverse force)
SMVFI	F_{I_z} (inertial component of vertical force)
SMVFD	F_{D_z} (drag component of vertical force)
SMVF	F_z (total vertical force)
SMHF	F_y (total transverse force)
SMVM	M_z (moment about y axis)
SMHM	M_y (moment about z axis)
RATIO	$\frac{\text{(drag component of vertical force)}}{\text{total vertical force}}$

The hyperbolic sine and cosine terms required in the wave equations were computed by statement number 1 in which

Program Symbols	Meaning in Theory Symbols (Explanation)	
A(1) = Kh	C(1,1) = sinh Kh	C(2,1) = cosh Kh
A(2) = 2Kh	C(1,2) = sinh 2Kh	C(2,2) = cosh 2Kh
A(3) = K(h+z)	C(1,3) = sinh K(h+z)	C(2,3) = cosh K(h+z)
A(4) = 2K(h+z)	C(1,4) = sinh 2K(h+z)	C(2,4) = sinh 2K(h+z)

The most illustrative way to show how symbols E(1,1), E(1,2) .... etc. are used is by rewriting the transverse and vertical velocity and acceleration equations with these symbols inserted.

Thus

$v = E(1,1)\sin\psi + F(1,1)\cos 2\psi + G(1,1)$ (see equation 28)

$w = F(2,2)\cos\psi + E(2,2)\sin 2\psi + G(2,2)$ (see equation 29)

$dv/dt = F(1,2)\cos\psi + E(1,2)\sin 2\psi + G(1,2)$ (see equation 30)

$dw/dt = E(2,1)\sin\psi + F(2,1)\cos 2\psi + G(2,1)$ (see equation 31)

By noting the E(M,N) terms are always multiplied by sine terms and the F(M,N) terms are always multiplied by cosine terms, I believe that the reader can work out statement number 4 (VIP(M,N) ....) to see that the vertical and transverse velocity and acceleration terms at a particular x(J) and a particular time (I) are as follows:

VIP(1,1) = v

VIP(1,2) = dv/dt

VIP(2,1) = dw/dt

VIP(2,2) = w

It should be noted that the above explanation of the symbols in the program should not be considered a complete analysis of the program. It is offered only as a guide to help those interested in following the programming procedure involved in the modified Morrison program.

```
C      R.J. ENGLERT  J.R.  THUNE  THESIS PROJECT
C      FORCES AND MOMENTS ON A BODY OF REVOLUTION IN WATER WAVES
C      THEORY BASED UPON THE SUPERPOSITION OF INERTIAL AND
C      DRAG COMPONENTS TO OBTAIN TOTAL FORCE
C
       DIMENSION C(4,2),X(7),VIP(2,2),D(7),RN(2,2),CD(2,2),CM(7,2)
       DIMENSION A(4),E(2,2),F(2,2),G(2,2),HH(2),DSQ(7)
   10  PAUSE
       ACCEPT,WL,BL,H,DEPTH,Z,RHO,VISC,ALPHA,S1,S2,S3,S4,S5,S6,S7
       B=6.2832/WL
       A(1)=B*DEPTH
       A(2)=2.*A(1)
       A(3)=A(1)+B*Z
       A(4)=2.*A(3)
       DO 1 I=1,4
       DO 1 J=1,2
    1  C(I,J)=.5*(EXP(A(I))+((-1.)**J)*1./EXP(A(I)))
       TANH=C(1,1)/C(1,2)
       PIHOL=3.1416*H/WL
       PIHSQ=PIHOL*PIHOL
       CSQ=C(1,1)**2
       ALSIN=SIN(ALPHA)
       ALCOS=COS(ALPHA)
       IF(C(1,1)-10.0E10)8,9,9
    9  CEL=SQR(32.174*TANH/B)
       F(1,1)=0.0
       E(2,2)=0.0
       ETACN=0.0
       GO TO 6
    8  SS1=PIHSQ*(5.+2.*C(2,2)+2.*C(2,2)**2)/(8.*CSQ**2)
       CEL=SQR(32.174*TANH/B*(1.+SS1))
       F(1,1)=-PIHSQ*CEL*ALSIN/CSQ*(-.5+.75*C(4,2)/CSQ)
       E(2,2)=-.75*PIHSQ*CEL*C(4,1)/(CSQ*CSQ)
       ETACN=-PIHOL*H*.25*C(1,2)*(2.+C(2,2))/CSQ**2
    6  SIGMA=B*CEL
       E(1,1)=PIHOL*CEL*C(3,2)*ALSIN/C(1,1)
       G(1,1)=PIHOL*CEL*ALSIN*.5*C(4,2)/CSQ
       F(2,2)=-PIHOL*CEL*C(3,1)/C(1,1)
       G(2,1)=0.0
       G(2,2)=0.0
       E(1,2)=-SIGMA*F(1,1)*2.
       F(1,2)=SIGMA*E(1,1)
       G(1,2)=0.0
       E(2,1)=-SIGMA*F(2,2)
       F(2,1)=SIGMA*E(2,2)*2.
       HH(1)=E(1,1)+G(1,1)
       HH(2)=-F(2,2)
       X(1)=-.44195*BL
       X(2)=-.26485*BL
       X(3)=-.16195*BL
       X(4)=0.0
```

```
      X(5)=-X(3)
      X(6)=-X(2)
      X(7)=-X(1)
      TIME=WL/CEL
      DO 12 J=1,7
      DIAM=S1+S2*X(J)+S3*X(J)**2+S4*X(J)**3+S5*X(J)**4
      D(J)=DIAM+S6*X(J)**5+S7*X(J)**6
      DSQ(J)=D(J)*D(J)
      DO 12 L=1,2
      CMCN=HH(L)*TIME/D(J)
      IF(CMCN-7.5)30,30,31
   30 CMCN1=2.22-7.2002670E-02*CMCN
      CMCN2=2.3201049E-02*CMCN**2-3.5200945E-03*CMCN**3
      GO TO 12
   31 IF(CMCN-20.)32,32,33
   32 CMCN1=8.0717750-1.6777438*CMCN+.14323456*CMCN**2
      CMCN2=-5.4474432E-03*CMCN**3+7.9358277E-05*CMCN**4
      GO TO 12
   33 CMCN1=7.36985E-02+5.1852715E-02*CMCN
      CMCN2=-4.3765884E-04*CMCN**2+1.5350526E-06*CMCN**3
   12 CM(J,L)=CMCN1+CMCN2
      DO 2 I=1,11
      PP=I
      T=.62832*(PP-1.)
      SMVFI=0.0
      SMVFD=0.0
      SMVM=0.0
      SMHF=0.0
      SMHM=0.0
      ETA=.5*(H*SIN(T)+ETACN*COS(2.*T))
      DO 3 J=1,7
      Y=B*X(J)*ALCOS+T
      DO 4 M=1,2
      DO 4 N=1,2
      P=N
    4 VIP(M,N)=E(M,N)*SIN(P*Y)+F(M,N)*COS(2.*Y/P)+G(M,N)
      DO 5 K=1,2
      RNCON=SQR(VIP(K,K)**2)*D(J)/VISC
      RN(K,K)=LOG(RNCON)/2.303
      IF(RN(K,K)-3.301)40,40,41
   40 CDC1=.97323833-.62865441*RN(K,K)+.1670641*RN(K,K)**2
      CDC2=-1.5636585E-02*RN(K,K)**3-1.8406664E-03*RN(K,K)**4
      GO TO 50
   41 IF(RN(K,K)-5.699)42,42,43
   42 CDC1=-1.2369606+3.6492456*RN(K,K)-2.2776174*RN(K,K)**2
      CDC2=.52382332*RN(K,K)**3-4.0687398E-02*RN(K,K)**4
      GO TO 50
   43 CDC1=-7.7171679+2.7592113*RN(K,K)
```

```
      CDC2=-.34163942*RN(K,K)**2+1.3958782E-02*RN(K,K)**3
   50 POP=CDC1+CDC2
    5 CD(K,K)=10.**(POP)
      VFI=CM(J,2)*.7854*DSQ(J)*VIP(2,1)*RHO*BL/7.
      VFD=CD(2,2)*RHO*.5*SQR(VIP(2,2)**2)*VIP(2,2)*D(J)*BL/7.
      HFI=RHO*CM(J,1)*.7854*DSQ(J)*VIP(1,2)*BL/7..
      HFD=CD(1,1)*.5*D(J)*SQR(VIP(1,1)**2)*VIP(1,1)*BL/7.
      HF=HFI+HFD
      SMVFI=SMVFI+VFI
      SMVFD=SMVFD+VFD
      SMVM=SMVM+(VFD+VFI)*X(J)
      SMHF=SMHF+HF
    3 SMHM=SMHM+HF*X(J)
      SMVF=SMVFI+SMVFD
      RATIO=SMVFD/SMVF
      PRINT,SMVF,SMHF,SMVM,SMHM,RATIO
    2 PRINT,ETA,CEL
      GO TO 10
      END
```

```
C       POTENTIAL FLOW SOLUTION FOR STOKIAN WAVE PROFILE
C       FORCES AND MOMENTS ON A SUBMERGED BODY OF REVOLUTION
C       R J ENGLERT  J R THUNE  THESIS
C
C
      DIMENSION A(4),C(4,2),E1(2),E2(2),E3(2),X(7),F(3)
      DIMENSION V(2),UT(3),Q1(2),Q2(2)
   10 PAUSE
      ACCEPT,WL,BL,H,DEPTH,Z,RHO,VISC,ALPHA,S1,S2,S3,S4,S5,S6,S7
      B=6.2832/WL
      A(1)=B*DEPTH
      A(2)=2.*A(1)
      A(3)=A(1)+B*Z
      A(4)=2.*A(3)
      DO 1 I=1,4
      DO 1 J=1,2
    1 C(I,J)=.5*(EXP(A(I))+((-1.)**J)*1./EXP(A(I)))
      TANH=C(1,1)/C(1,2)
      PIHOL=3.1416*H/WL
      PIHSQ=PIHOL*PIHOL
      ALSIN=SIN(ALPHA)
      ALCOS=COS(ALPHA)
      IF(C(1,1)-10.E10)8,8,9
    9 CEL=SQR(32.174*TANH/B)
      P3=0.0
      ETACN=0.0
      GO TO 6
    8 SS1=PIHSQ*(5.+2.*C(2,2)+2.*C(2,2)**2)/(8.*C(1,1)**4)
      CEL=SQR(32.174*TANH/B*(1.+SS1))
      ETACN=-PIHOL*H*.25*C(1,2)*(2.+C(2,2))/C(1,1)**4
      P3=-PIHSQ*CEL*.75/C(1,1)**4
    6 SIGMA = B*CEL
      P1=PIHOL*CEL/C(1,1)
      P2=PIHSQ*CEL*.5/C(1,1)**2
      P4=P2
      E1(1)=SIGMA*C(3,1)
      E2(1)=0.0
      E3(1)=2.*SIGMA*C(4,1)
      E1(2)=C(3,2)*SIGMA
      E2(2)=-SIGMA*2.
      E3(2)=-C(4,2)*SIGMA*2.
```

```
      X(1)=-.44195*BL
      X(2)=-.26485*BL
      X(3)=-.16195*BL
      X(4)=0.0
      X(5)=-X(3)
      X(6)=-X(2)
      X(7)=-X(1)
      CC=-12.5664*RHO
      DO 30 I=1,11
      PP=I
      T=.62832*(PP-1.)
      ETA=.5*(H*SIN(T)+ETACN*COS(2.*T))
      DO 3 M=1,3
    3 F(M)=0.0
      VM=0.0
      HM=0.0
      DO 20 J=1,7
      PSI=B*X(J)*ALCOS+T
      Q1(1)=SIN(PSI)
      Q2(1)=COS(2.*PSI)
      Q1(2)=COS(PSI)
      Q2(2)=SIN(2.*PSI)
      DO 4 K=1,2
    4 V(K)=P1*E1(K)*Q1(K)+(P2*E2(K)+P3*E3(K))*Q2(K)
      DIAM=S1+S2*X(J)+S3*X(J)**2+S4*X(J)**3+S5*X(J)**4
      D=DIAM+S6*X(J)**5+S7*X(J)**6
      UT(1)=-V(2)*ALCOS*D**2/16.
      UT(2)=-V(2)*ALSIN*D**2/8.
      UT(3)=-V(1)*D**2/8.
      DO 40 M=1,3
   40 F(M)=CC*UT(M)*BL/7.+F(M)
      VM=CC*UT(3)*X(J)*BL/7.+VM
   20 HM=CC*UT(2)*X(J)*BL/7.+HM
      PRINT,F(3),VM,F(2),HM,F(1)
   30 PRINT,ETA
      GO TO 10
      END
```

```
C       R J ENGLERT   J R THUNE THESIS PROJECT
C       POTENTIAL FLOW SOLUTION FOR SINE WAVE PROFILE
      DIMENSION X(7),D(7),DSQ(7)
   10 PAUSE
      ACCEPT,WL,BL,H,DEPTH,Z,RHO,VISC,ALPHA,S1,S2,S3,S4,S5,S6,S7
      B=6.2832/WL
      ALSIN=SIN(ALPHA)
      ALCOS=COS(ALPHA)
      CEL=SQR(5.12062*WL)
      X(1)=-.44195*BL
      X(2)=-.26485*BL
      X(3)=-.16195*BL
      X(4)=0.0
      X(5)=-X(3)
      X(6)=-X(2)
      X(7)=-X(1)
      TIME=WL/CEL
      PTCON=B*RHO*25.26632*H*EXP(B*Z)
      DO 12 J=1,7
      DIAM=S1+S2*X(J)+S3*X(J)**2+S4*X(J)**3+S5*X(J)**4
      D(J)=DIAM+S6*X(J)**5+S7*X(J)**6
   12 DSQ(J)=D(J)**2
      DO 2 I=1,11
      PP=I
      T=.62832*(PP-1.)
      SMPVF=0.0
      SMPVM=0.0
      SMPHF=0.0
      SMPHM=0.0
      ETA=.5*H*SIN(T)
      DO 3 J=1,7
      Y=B*X(J)*ALCOS+T
      SMPHF=SMPHF+PTCON*ALSIN*DSQ(J)*COS(Y)*BL/7.
      SMPHM=SMPHM+PTCON*ALSIN*DSQ(J)*X(J)*COS(Y)*BL/7.
      SMPVF=SMPVF+PTCON*DSQ(J)*SIN(Y)*BL/7.
    3 SMPVM=SMPVM+PTCON*DSQ(J)*X(J)*SIN(Y)*BL/7.
      PRINT,SMPVF,SMPHF,SMPVM,SMPHM
    2 PRINT,ETA
      IF(SENSE SWITCH 1)20,21
   20 PRINT,Y,PTCON,T
   21 PRINT,CEL
      GO TO 10
      END
```

APPENDIX D. DESIGN OF TORSION GAGE

A. General

A particular array of strain gages mounted on a thin walled cylindrical tube was found to be an effecient and simple design for the torsion gage (14). The arrangement of the strain gages is illustrated in Fig. 23.

Configuration of gages

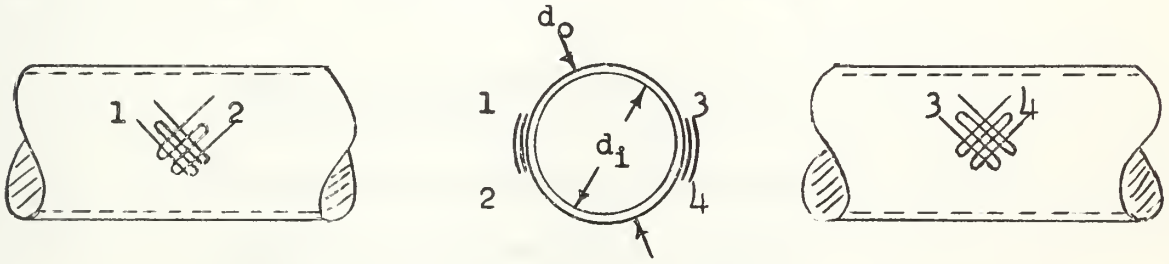
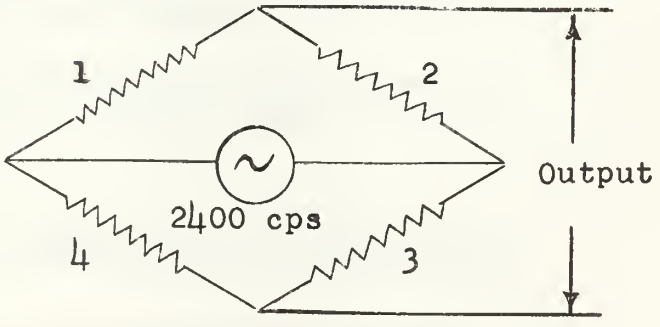


Figure 23



Circuit Diagram

Figure 24

The configuration shown utilizes the fact that the principal strain axes of a shaft in torsion are 45° from the circumferential reference line. Note also that the magnitude of the principal compressive strain is equal to the magnitude of the principal tensile strain. By using the principle of the wheatstone bridge which states that the output of opposite arms of a strain gage bridge are additive and adjacent arms subtractive, the gages were connected as indicated in Fig. 24 for maximum sensitivity.

The characteristics of the torsion gage described above are as follows:

1. Output is proportional to the applied torque.
2. Output is independent of bending.
3. Output is independent of axial thrust.
4. As long as temperature changes take place uniformly throughout the tube, the output is independent of temperature changes.
5. The total output is four times the output of a single gage.

B. Calculation of Gage Size and Sensitivity

From the basic relationship of solid mechanics between stress and torsional moment applied to a shaft or cylindrical tube:

$$\tau = \frac{M_y \, d_{o/2}}{I_p} \tag{46}$$

in which τ = shear stress at the outer surface of the tube or shaft.

d_o = outside diameter.

M_y = moment in the horizontal plane (torque applied)

I_p = polar moment of inertia of the section.

$$I_p = \frac{\pi}{32} d^4 \quad \text{for solid shaft}$$

$$I_p = \frac{\pi}{32} (d_o^4 - d_i^4)$$

Substituting the basic stress strain relationship:

$$\tau = \gamma G$$

in which γ = shear strain at the surface

G = modulus of elasticity in shear (modulus of rigidity)

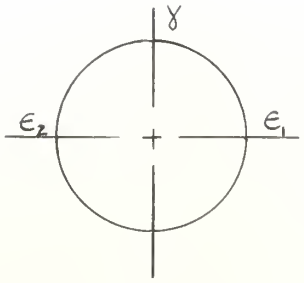
Equation 46 then beomes:

$$\gamma G = M_y \frac{d_{o/2}}{I_p} \tag{47}$$

or

$$M_y = \frac{I_p G}{d_o} \, 2\gamma$$

From an investigation of Mohrs circle for the torsional case



in which: ϵ_1 = Principal tensile strain

ϵ_2 = Principal compressive strain

it can be seen that $\gamma = \frac{\epsilon_1 - \epsilon_2}{2}$

substituting γ into 47:

$$M_y = \frac{I_p G}{d_o} (\epsilon_1 - \epsilon_2) \qquad (48)$$

Also from the Mohrs circle:

$\epsilon_2 = -\epsilon_1$ (Maximum compressive strain is equal in magnitude but opposite in sign to the maximum tensile strain)

then

$$M_y = \frac{I_p G}{d_o} (2 \epsilon_1) \qquad (49)$$

The above equation is applicable to two strain gages located on one side of the gage. By adding the output of the other two gages the torque equation becomes:

$$M_y = \frac{I_p G}{d_o} (4 \epsilon_1) \qquad (50)$$

Substituting $G = \frac{E}{2(1 + \mu)}$ into equation (50) results in

$$M_y = \frac{E}{2(1 + \mu)} \frac{I_p}{d_o} (4 \epsilon_1) \qquad (51)$$

Because of difficulties encountered in bonding strain gages to small radius objects, a 1-inch outside diameter tubing was considered to be the minimum practical size.

The Sanborn recorder instruction manual indicated the maximum sensitivity of a 4 active arm bridge to be about 10 micro inches/inch/ centimeter deflection of the Sanborn stylus. Therefore, by substituting this value for ϵ_1 in equation (51) and the value of I_p corresponding to a one inch outside diameter, .035 inch wall thickness aluminum tubing, (smallest commercial size available), the torque necessary to produce one centimeter of stylus deflection can be calculated. Substituting in equation (50) the following values:

$$E = 10 \times 10^6 \text{ for aluminum}$$

$$\epsilon_1 = 10 \times 10^{-6}$$

$$\mu = .33$$

$$d_o = 1.0$$

$$I_p = .0247 \text{ in}^4$$

then

$$M_y = \frac{(10\text{x}10^6)(40\text{x}10^{-6})(.0247)}{2(1 + .33)(1.0)}$$

$$M_y = 3.71 \text{ inch - lbs/cm}$$

or

$$M_y = .309 \text{ ft.-lbs/cm}$$

This figure agrees very well with the actual calibration trace of the torsion gage which yielded .32 ft.-lbs/cm.

The strain gages used for this instrument were Tatnall foil gages, type 121-R3A.

Eastman 910 cement was used to bond the gages to the tubing. The area in which the gages were located was thoroughly painted with TMS-GW-1 type waterproofing in order to waterproof the gages and to hold the fragile wiring in place. The gage itself proved to be very effective in measuring the moment in the horizontal plane about the vertical axis of the body.

Although the torsion gage lowered the natural frequency of the overall gage system slightly in bending, the natural frequency of the system in torsion was about 30 cps, thus being well out of the range of the forcing frequencies used.

thesE46
Wave forces and moments on a submerged e
3 2768 002 06186 3
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